# Tensile Fatigue Capacity of Concrete

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## ABSTRACT

Results and analyses are presented from cyclic uniaxial tensile tests on plain cylindrical concrete cores. The influence of the load amplitude and the mean load level were studied with so called factorial design. It was found that both factors were important but that neither of them could be established to be more important than the other. Further, the deformation rate was studied. It appears that a certain fatigue limit exists below which a clearly greater number of load cycles is required for failure. From this research the exact limit cannot be predicted, but for tests with a mean load level of 40% of $f_{peak}$ and an amplitude of 40% of $f_{peak}$, a very low deformation rate has been obtained. Finally, the test results have been compared with other Wöhler curves proposed for cyclic load in tension.

**Keywords**: concrete, fatigue capacity, tensile strength, Wöhler curve, deformation rate, uniaxial tensile test.

## 1 INTRODUCTION

During the last few years the concrete fatigue phenomenon has once again gained interest, especially for railway bridges due to more slender structures, higher traffic speeds and higher axle loads. In Sweden for example, the increased axle loads on the existing railway lines have caused problems with the bridges since it has led to a change of the conditions for the bridges compared to the ones when they were built. One of the problems is that the bridges often are predicted to fail in fatigue (e.g. shear fatigue failure) when they are evaluated with the present concrete codes.

In the tensile fatigue tests performed in this paper, the following three subjects have been studied:

- Influencing factors: Which of the load amplitude and the mean load level has the highest influence on the fatigue capacity, i.e. the number of load cycles to failure? This will be examined using so-called factorial design.

- Deformation rate: Could the deformation rate in a fatigue test give any special information regarding the fatigue capacity? Is there a load level limit, below which no fatigue failure occurs?

- Wöhler curve: How do the results compare to earlier Wöhler curves for tension presented in the literature?

## 2 EXPERIMENTAL SET-UP

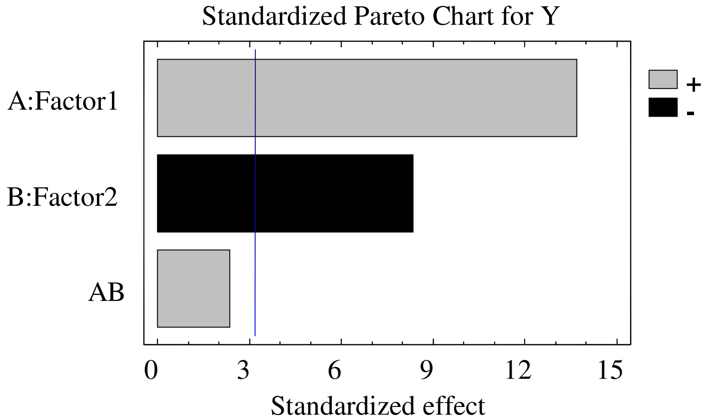
### 2.1 Experimental design – factorial design

There are several experimental strategies that can be used when planning a test series. In Montgomery[1] factorial design is described as a strategy where the involved factors are varied together instead of e.g. one at a time. One big advantage is that it considers interactions between the factors. Factorial design is in other words said to be a suitable method to examine if a factor has an influence on a specific variable or not. Montgomery writes that factorial design means that in each complete replication of the experiment all possible combinations of the studied levels of the examined factors are investigated. Montgomery exemplifies it as: if there are $a$ levels of factor $A$ and $b$ levels of factor $B$, each replicate contains all $ab$ treatment combinations. When factors are arranged in factorial design they are often said to be crossed. Montgomery further writes that the effect of a factor is defined to be a change in the response produced by a change in the level of the factor. Since this refers to the primary factors of interest in the experiment, it is often called a main effect. In a two-factor factorial experiment the levels are denoted with low (-) and high (+), this could also be written as a "$2^2$-factorial design" and in a more general form "$2^k$-factorial design", where the "2" is the number of levels and the "$k$" represents the number of factors. In an analysis it is also assumed that the factors are fixed, the design is randomised and the factors are normally distributed. Often statistic software is used to set-up and analyse $2^k$-factorial designs. In this analysis the computer software Statgraphics (by Statistical Graphics Corp.) has been used.

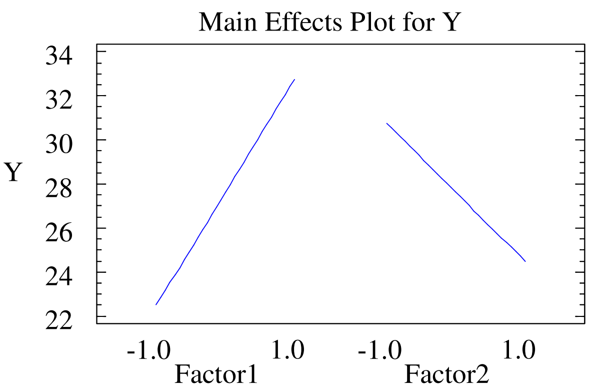
In Figure 1 the method is explained with an example. In Figure 1a the test matrix is shown. The example is a $2^2$ factorial design that consists of two factors, 1 and 2, varied at two levels, high and low. The tests are replicated twice and the response is called $Y$. In Figure 1b to Figure 1d the results are presented from an analysis with the help of the software Statgraphics. In Figure 1b a so-called Pareto chart, a bar chart, is presented where each factor is represented with a horizontal bar. There is also a vertical line that is used to test the significance of the effect, here the significance level, $\alpha$, equal to 5% has been chosen. If any bar stretches beyond this line the factor has a significant influence on the result. In this case both factor 1 and 2 separately have a significant influence on the result but they are independent of each other (no interaction). In a main effects plot, see Figure 1c, the effect on the response $Y$ from each tested factor is shown. It can be seen from the example that a lower value of factor 1 gives a lower value of $Y$ than keeping it at a high level. In Figure 1d the interaction plot is shown, where the response variable for each combination of factor 1 and 2 is shown. From the example it is shown that there is no interaction between the factors (the two lines would then cross each other). The example gives that if factor 1 is high and factor 2 is low it results in a $Y$ equal to 35. If factor 1 is kept at a high level and factor 2 is also high it results in a $Y$ equal to 30.

| Run | Factor | | Treatment | Response |
|---|---|---|---|---|
| | One | Two | combination | |
| 1 | + | - | 1 high, 2 low | 37 |
| 2 | - | + | 1 low, 2 high | 19 |
| 3 | + | + | 1 high, 2 high | 32 |
| 4 | - | - | 1 low, 2 low | 27 |
| 5 | + | + | 1 high, 2 high | 29 |
| 6 | - | - | 1 low, 2 low | 26 |
| 7 | + | - | 1 high, 2 low | 33 |
| 8 | - | + | 1 low, 2 high | 18 |

*a)*




*b)*

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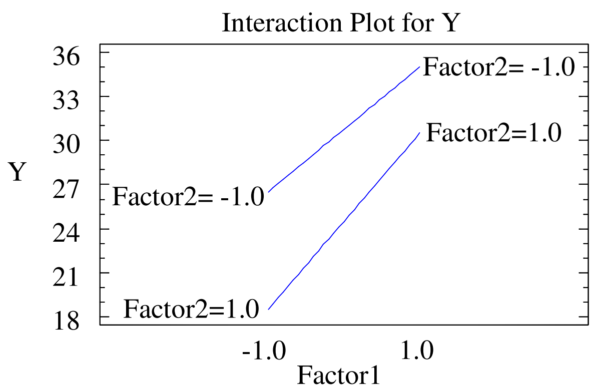



*d)*

*Figure 1 - Factorial design according to Montgomery[1]. a) Example matrix, b) Results from analysis in Statgraphics – Pareto chart, c) Results from analysis in Statgraphics – Main effects plot and d) Results from analysis in Statgraphics – interaction plot.*

In this study two different factors that influence the fatigue capacity have been compared, i.e. the load amplitude and the mean load level, hopefully to see which of them is the most important parameter. In Figure 2 the experimental design is shown. Two different mean load levels (40% and 60% of $F_{peak}$) have been tested with two different load amplitudes (40% and 60% of $F_{peak}$). The limit of maximum load, $F_{max}$ (or $f_{max}$ if the stress is used) for the tests have been set to 90 % of the mean peak load, $F_{peak}$ (or $f_{peak}$ if the stress is used) from the static uniaxial tests.

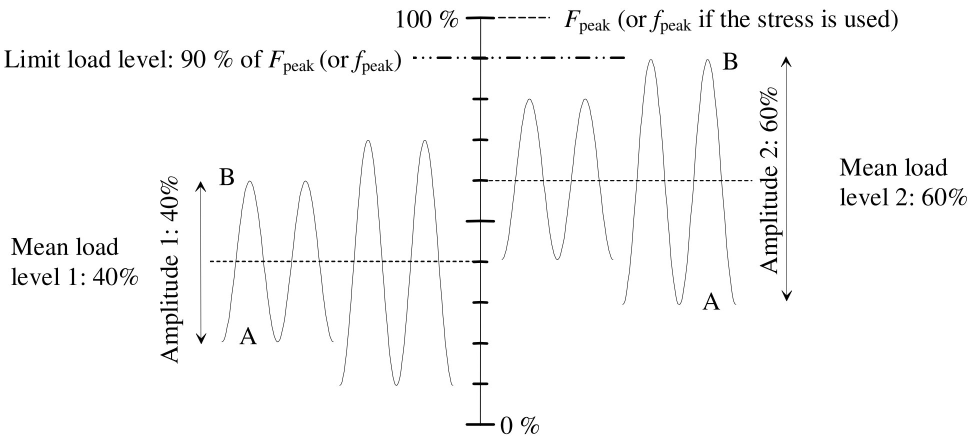



*Figure 2 - Visualisation of the experimental design used in the fatigue tests.*

## 2.2 Specimen type, dimensions and methods

In order to determine the strength of the concrete (so that the load levels in the fatigue test could be set) eight uniaxial tensile tests were performed under displacement control using a closed-loop servo-hydraulic test machine. A total of four Crack Opening Displacement gauges (COD-gauges) have been used to measure the deformation and the feed-back signal to the machine was the mean value of all four COD-gauges, see Figure 3.

All tests have been performed on drilled cores with a height and a diameter of approximately 100 mm. The cores were drilled from small slabs cast in March 2004. Three days before testing the drilled cores were cut into the test length of about 100 mm, a notch was milled (leaving a diameter of about 74 mm) and they were then air-cured in the laboratory at room temperature until the testing day. For dimensions see Figure 3. Today there is no international standard on how to perform a uniaxial tensile (fatigue) test. The influence of the shape and the dimension etc. of the specimen has been studied by several researchers, e.g. Daerga[2], Hordijk[3] and Noghabai[4]. In Noghabai for instance, different shapes/radii of the notch were studied and the conclusion was that the differences most likely were within the normal scatter in the experiments, implying that normal concrete is fairly notch insensitive.

The concrete used was a normal strength concrete (NSC) with a maximum aggregate size of 16 mm. In a uniaxial test it is important that the aggregate size is not too big in proportion to the fracture area, which imply that it is important to study the failure surface after a test. In Hordijk[5] it is recommended that the diameter of the fracture area should be 4 to 5 times the aggregate size. The concrete was designed to have a characteristic compressive strength of 45 MPa, tested on 150 mm cubes after 28 days (according to the Swedish concrete recommendation, BBK94[6]).

All fatigue tests have been performed under load control with sinusoidal load cycles. The load frequency has been 2.0 Hz. The analysis of the data has been performed with the computer programme MATLAB™ (the MathWorks Inc.).

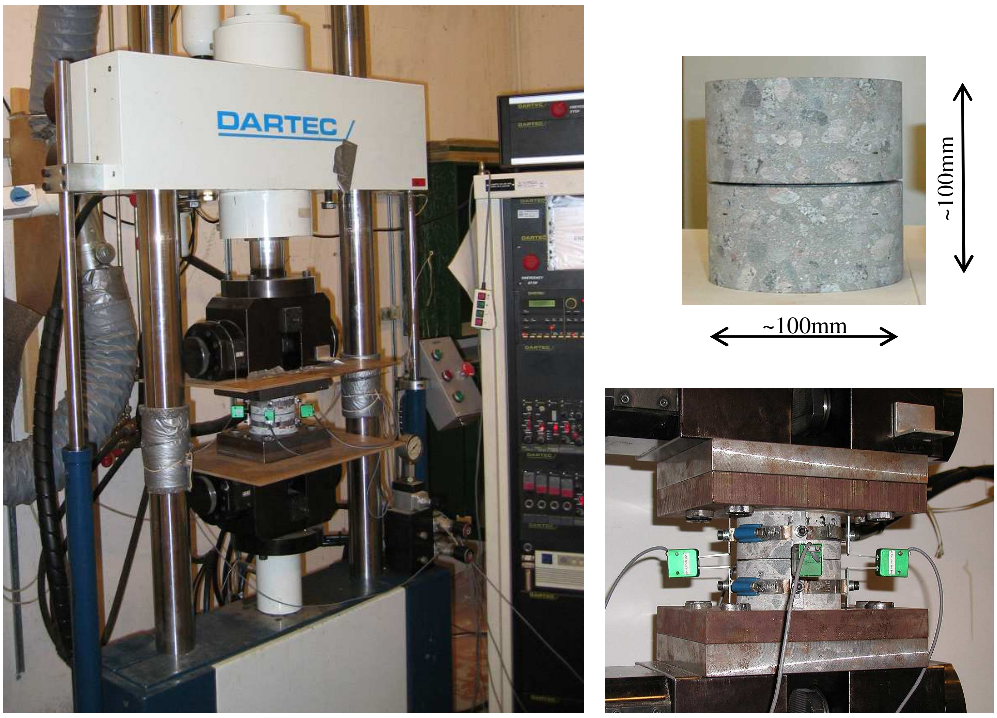



*Figure 3 - Left: Servo hydraulic machine and other equipment used in the tests. Above to right: Photo of a specimen, from Andersson*[7]*. Below to the right: Photo showing the specimen mounted in the machine and the used COD-gauges.*

The growth in deformation during a fatigue test can be divided into three phases, see Figure 4. At the beginning of the first phase the deformation rate is high but stagnates after a while. The second phase is characterised by a constant deformation rate. These two phases can be described as stable. During the third phase, the failure phase, the deformation rate increases rapidly leading to failure within a short time. A number of parameters have been determined from the fatigue curves and they are visually shown in Figure 4:

- $n_{1-2}^{\mathrm{U}}$ and $\delta_{1-2}^{\mathrm{U}}$, are the number of load cycles and the deformation respectively, at the point on the upper fatigue curve where phase 1 ends and phase 2 starts. $n_{1-2}^{\mathrm{L}}$ and $\delta_{1-2}^{\mathrm{L}}$ is the same point on the lower curve.

- $n_{2-3}^{\mathrm{U}}$ and $\delta_{2-3}^{\mathrm{U}}$, define the point on the upper fatigue curve where phase 2 ends and phase 3 starts and $n_{2-3}^{\mathrm{L}}$ and $\delta_{2-3}^{\mathrm{L}}$ define the same point on the lower curve.

- $\delta_{\alpha}^{\mathrm{U}}$ and $\delta_{\alpha}^{\mathrm{L}}$ are the deformation rate for the upper and lower fatigue curve respectively during phase 2 [mm/cycle].

- $\delta_{1-2}^{\mathrm{A}}$ is the deformation amplitude at the point where phase 1 changes to phase 2 and $\delta_{2-3}^{\mathrm{A}}$ is the deformation amplitude at the point where phase 2 changes to phase 3.

- $\delta_{\max}^{\mathrm{A}}$ is the maximum amplitude measured during the fatigue test, often at the very end of the test.

- $\delta_{ul}$ is defined as the highest deformation measured during the fatigue test for a complete cycle.

The inflection points, e.g. the point ($n^{U}_{1-2}$, $\delta^{U}_{1-2}$), have been determined in the following way. With the help of the Matlab™, a linear equation has been fitted to the test data for phase 2 which gives the slope $\delta^{U}_{\alpha}$. This linear equation has in turn been compared to the measurement data and where the difference between the linear equation and the measurement data, is larger than the deformation rate, $\delta^{U}_{\alpha}$, multiplied with a load cycle increment, $\Delta n$ (individual for each test), an inflection point has been found. For the tests which have lasted for a short time a load cycle increment, $\Delta n$, of 0.01 has been used. This low increment has not been possible to use for the longer fatigue tests where min-max sampling has been used, due to the fact that the data scatter more. This definition and method of determining the inflection points are not exact. However, the method gives an approximation that is satisfactory, since the increase in deformation is small for the tests that last for more than approximately 10000 cycles.

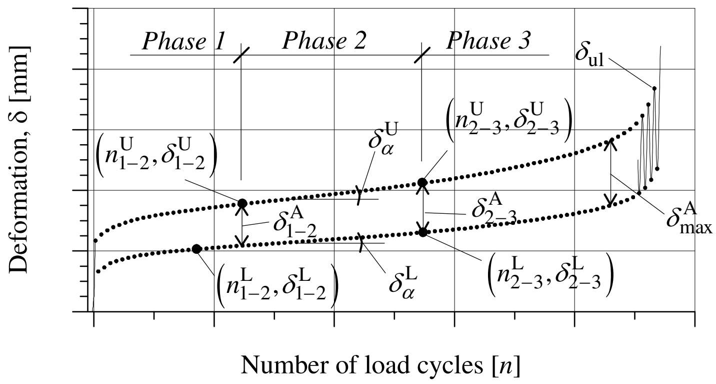



*Figure 4 – Graph showing, in principal, the definition of parameters from fatigue tests.*

### 2.3 Sampling of data

The intention was to sample the data from the COD-gauges in the fatigue tests continuously with a frequency of 60 Hz. However, this leads to very large data files which are difficult to handle. This problem has led to the use of a measuring technique, here called min-max-sampling, where only the maximum and minimum deformations for a time period that each lasts 1.5 seconds have been saved (together with the maximum time value for the same period), see Figure 5. The maximum and the minimum values are mean values of the four COD-gauges respectively. This technique results in smaller data files and the possibility of measuring without saving to a file, for approximately 17 days. The disadvantage with the technique is that the precision becomes somewhat lower at the start and at the end of each fatigue test. This was partly solved by sampling the start of each fatigue test with 60 Hz and when it was assessed that phase 2 was reached, i.e. a constant deformation rate was obtained, the sampling was changed to min-max-sampling. The intentions were then, when phase 3 was reached i.e. the failure phase, to switch back to sampling with 60 Hz. This was not practically possible since the time period of this phase could be so short that there was not enough time to make the switch. So, min-max-sampling was kept until the test was finished. The technique is not a perfect solution but the accuracy was under the circumstances satisfactory.

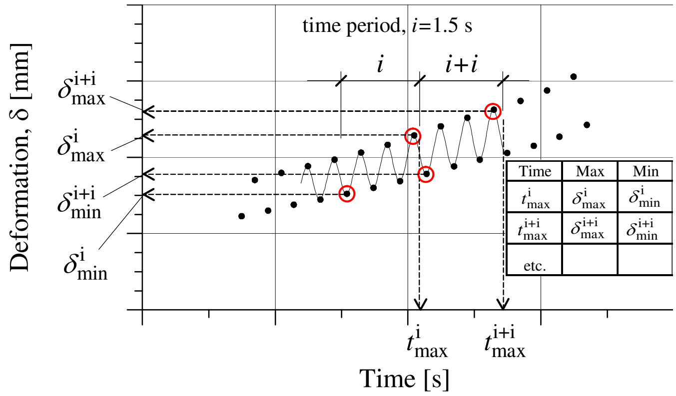



*Figure 5 - Description of the developed measuring technique where the maximum and minimum deformations for a time period of 1.5 seconds are stored together with the maximum time value during the same period.*

# 3 RESULTS

## 3.1 General

The compressive concrete strength was tested on 150 mm cubes in October 2004 (6 months after casting). After casting the small slabs were stored in a water tank (cores were drilled before the tests and stored together with the small slabs). Three days before testing the drilled cores were cut into the test length of about 100 mm, a notch was milled and they were then air-cured in the laboratory at room temperature until the testing day.

The mean concrete compressive cube strength was 72.2 MPa with a standard deviation of 1.9 MPa and a coefficient of variation of 3 %.

The splitting strength tested on similar cubes was 5.5 MPa with a standard deviation of 0.04 MPa and a coefficient of variation of 1 %. The corresponding uniaxial tensile strength becomes 4.4 MPa (if the Swedish concrete recommendations, BBK94[6] is used, where the uniaxial tensile strength is set to 80% of the splitting strength and in the EN 1992-1-1[8] the uniaxial tensile strength is set to 90% of the splitting strength).

The mean uniaxial tensile strength for the eight tests performed was 3.0 MPa (13.5 kN) with a standard deviation of 0.2 MPa (coefficient of variation 6.5%) and the individual values were 2.77, 3.29, 3.24, 2.92, 2.9, 2.94 and 3.17 MPa. This mean value is considerably lower than what was obtained from the splitting tests (4.4 MPa). The mean *E*-modulus was 32.2 GPa (derived from static uniaxial tension tests i.e. the tension modulus) for the used concrete mix, with a standard deviation of 2.6 MPa (coefficient of variation 8%).

## 3.2 Fatigue tests

In Table 1 the results from the fatigue tests are presented. Note that $f_{max}$ is 90 % of $f_{peak}$, which is the mean uniaxial tensile strength of 8 tests, see Figure 2. Test no. 20 was stopped at 5 million load cycles and a uniaxial tensile test was performed which first resulted in a failure at the adhesive layer. The specimen was then cut again to remove the old adhesive and a new uniaxial tensile test was performed. This time the failure happened in the milled notch with a tensile strength of 2.37 MPa as a result.

*Table 1 - Results from fatigue tests. Plus-sign indicates high level (60%) and minus-sign indicates low level (40%), according to factorial design. A, B and MLL (Mean Load Level) see definitions in Figure 2 and results in Appendix A in Thun[9].*

| **Test** | **Load levels % of $f_{peak}$** | | | | **Factorial design** | | **Actual loads [kN]** | | | **Run order** | $R$ $(\sigma_A/\sigma_B)$ | **No. load Cycles** |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **no.** | A | B | Amp. | MLL | Amp. | MLL | A | B | Amp. | | | $N_{max}$ |
| 2 | 30 | 90 | 60 | 60 | + | + | 4.0 | 12.1 | 8.1 | 8 | 0.33 | 96 |
| 12 | 40 | 80 | 40 | 60 | - | + | 5.4 | 10.7 | 5.3 | 3 | 0.5 | 227 283 |
| 16 | 20 | 60 | 40 | 40 | - | - | 2.7 | 8.0 | 5.3 | 6 | 0.33 | 623 683 |
| 17 | 20 | 60 | 40 | 40 | - | - | 2.7 | 8.0 | 5.3 | 1 | 0.33 | 1 350 166 |
| 20 | 20 | 60 | 40 | 40 | - | - | 2.7 | 8.0 | 5.3 | 7 | 0.33 | 5 000 000 [a)] |
| 25 | 10 | 70 | 60 | 40 | + | - | 1.3 | 9.4 | 8.1 | 5 | 0.13 | 132 645 |
| 28 | 30 | 90 | 60 | 60 | + | + | 4.0 | 12.1 | 8.1 | 12 | 0.33 | 14 |
| 30 | 40 | 80 | 40 | 60 | - | + | 5.4 | 10.7 | 5.3 | 2 | 0.5 | 20 |
| 32 | 40 | 80 | 40 | 60 | - | + | 5.4 | 10.7 | 5.3 | 9 | 0.5 | 1659 |
| 33 | 10 | 70 | 60 | 40 | + | - | 1.3 | 9.4 | 8.1 | 11 | 0.5 | 2661 |
| 34 | 10 | 70 | 60 | 40 | + | - | 1.3 | 9.4 | 8.1 | 10 | 0.13 | 121 518 |
| 35 | 30 | 90 | 60 | 60 | + | + | 4.0 | 12.1 | 8.1 | 4 | 0.33 | 60 |

*[a)] The test was stopped at 5 million load cycles and a static uniaxial test was performed.*

In Table 2 different parameters that are of interest from the fatigue curves are summarised. For tests no. 16, 17 and 20 none of the parameters defined in Figure 4 (except $N_{max}$) has been possible to determine due to the fact that these tests have been strongly affected by a temperature variation during the tests. The definitions of the parameters are given in Figure 4.

*Table 2 - Selected parameters from fatigue tests. Definitions are given in Figure 4 and results in Appendix A in Thun[9].*

| **Test no.** | **Level** | **Amp.** | $n^U_{1-2}$ [cycles] | $\delta^U_{1-2}$ [mm] | $n^U_{2-3}$ [cycles] | $\delta^U_{2-3}$ [mm] | $\delta^U_{\alpha}$ [$\cdot 10^{-3}$, mm/cycle] | $\delta_{ul}$ [mm] | $\delta^A_{max}$ [mm] | $N_{max}$ [cycles] |
|---|---|---|---|---|---|---|---|---|---|---|
| 30 | + | - | 7.33 | 0.0099 | 14.2 | 0.0137 | 0.5512 | 0.0204 | 0.0069 | 20 |
| 32 | + | - | 181.8 | 0.0055 | 428.8 | 0.0060 | 0.0017 | 0.0201 | 0.0063 | 1 659 |
| 12 [a)] | + | - | 4175 | 0.0034 | 176090 | 0.0033 | [b)] | 0.0099 | 0.0038 | 227 283 |
| 35 | + | + | 13.38 | 0.0066 | 41.2 | 0.0092 | 0.0969 | 0.0156 | 0.0071 | 60 |
| 2 | + | + | 17.23 | 0.0084 | 49.8 | 0.0103 | 0.0560 | 0.0305 | 0.0101 | 96 |
| 28 | + | + | 1.67 | 0.0075 | 8 | 0.0105 | 0.4740 | 0.0162 | 0.0073 | 14 |
| 33 | - | + | 346.5 | 0.0073 | 1531 | 0.0090 | 0.0014 | 0.0215 | 0.0103 | 2 661 |
| 34 [a)] | - | + | 29030 | 0.0040 | 85460 | 0.0044 | $7.55\cdot 10^{-6}$ | 0.0142 | 0.0096 | 121 518 |
| 25 [a)] | - | + | 99060 | 0.0081 | 114330 | 0.0089 | $5.47\cdot 10^{-5}$ | 0.0209 | 0.0107 | 132 645 |

*[a)] Min-max-sampling has been used to measure the deformation. [b)] The deformation rate has been almost zero. This test has partly been affected by the temperature/ moisture variation.*

| Test no. | Level | Amp. | $n^{L}_{1-2}$ [cycles] | $\delta^{L}_{1-2}$ [mm] | $n^{L}_{2-3}$ [cycles] | $\delta^{L}_{2-3}$ [mm] | $\delta^{L}_{\alpha}$ [$\cdot 10^{-3}$, mm/cycle] | $\delta^{A}_{1-2}$ [mm] | $\delta^{A}_{2-3}$ [mm] | $N_{max}$ [cycles] |
|---|---|---|---|---|---|---|---|---|---|---|
| 30 | + | - | 7.63 | 0.0076 | 14.71 | 0.0107 | 0.4467 | 0.0019 | 0.0032 | 20 |
| 32 | + | - | 167.7 | 0.0036 | 461.8 | 0.0041 | 0.0016 | 0.0019 | 0.0019 | 1 659 |
| 12 [a)] | + | - | 2490 | 0.0020 | 178620 | 0.0017 | [b)] | 0.0014 | 0.0016 | 227 283 |
| 35 | + | + | 13.71 | 0.0039 | 44.63 | 0.0059 | 0.0659 | 0.0027 | 0.0036 | 60 |
| 2 | + | + | 19.8 | 0.0053 | 49.14 | 0.0063 | 0.0348 | 0.0033 | 0.0039 | 96 |
| 28 | + | + | 4.97 | 0.0056 | 7.67 | 0.0065 | 3.2214 | 0.0032 | 0.0039 | 14 |
| 33 | - | + | 313.2 | 0.0037 | 1625.8 | 0.0050 | 0.0010 | 0.0035 | 0.0040 | 2 661 |
| 34 [a)] | - | + | 85560 | 0.0015 | 3020 | 0.0015 | $1.30\cdot10^{-6}$ | 0.0026 | 0.0030 | 121 518 |
| 25 [a)] | - | + | 105970 | 0.0044 | 114540 | 0.0046 | $2.58\cdot10^{-5}$ | 0.0041 | 0.0044 | 132 645 |

*[a)] Min-max-sampling has been used to measure the deformation. [b)] The deformation rate has been almost zero. This test has partly been affected by the temperature/ moisture variation.*

In Figure 6, Figure 7 and Figure 8 examples of results from the cyclic tensile fatigue test are presented, in this case specimens no. 25, 34 and 35. The diagrams illustrate the development of the mean deformation for the four COD-gauges. For explanation of the dots in Figure 6, Figure 7 and Figure 8 see the definitions in Figure 4.

If Figure 6 and Figure 8 (and Figure 7) are compared, it is seen that for test no. 25 phase 2 is repeated see Figure 6. Between these two phases 2 there is an unstable phase, called Z in Figure 6. This behaviour resembles of the case where an increase in load is done during a fatigue test, see Thun[10]. In this case a plausible reason for the repetition of phase 2, is that the propagating crack is temporarily hindered at an aggregate and after a while when it is stabilised the crack continues to propagate.

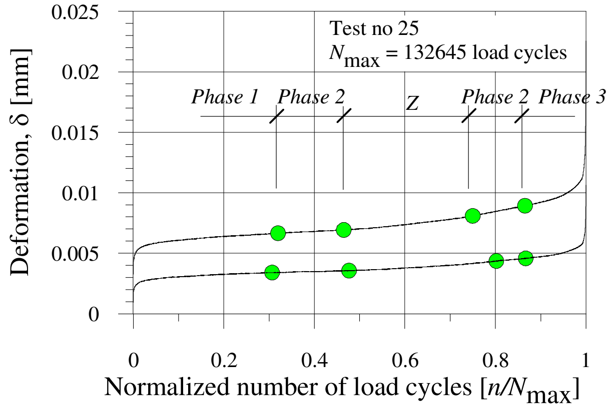



*Figure 6 - Result from the cyclic tensile fatigue test of specimen no. 25.*

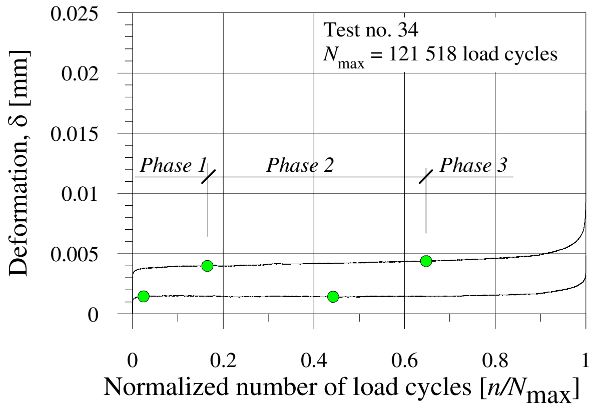



*Figure 7 - Result from the cyclic tensile fatigue test of specimen no. 34.*

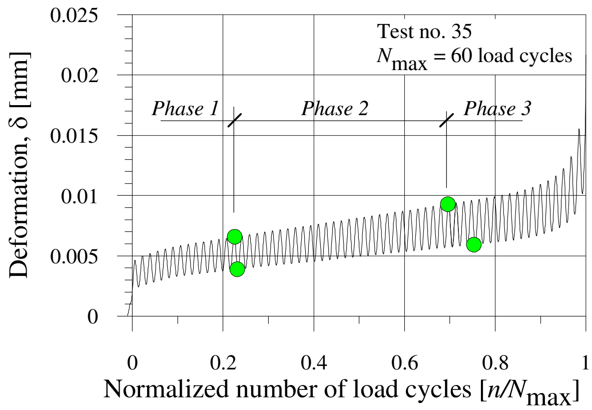



*Figure 8 - Result from the cyclic tensile fatigue test of specimen no. 35.*

# 4 ANALYSIS AND DISCUSSION

## 4.1 Uncertainties

There are several factors that influence the accuracy of the measured values. Such factors of importance are variations in the electric current and the temperature, deformations due to shrinkage and creep and the normal scatter in results that are present in long-time fatigue tests. The influence of these factors is estimated and discussed below.

- **Temperature**. The solution of the problem with the temperature turned out to be more complicated than believed. The best solution seemed to be to seal the ventilation as much as possible and to measure a “trend curve” before each fatigue test. This would result in a temperature curve that could be used to compensate the tests for the temperature deviation. This curve could of course only be used if the tests did not last too long and there were no dramatic changes in the temperature during this period of time. Unfortunately, it turned out that this method could not be used in this test series due a to big difference between the measured temperature-trend-curve a few hours before the tests started and the conditions during the actual tests (tests with no. 16, 17 and 20).

It is difficult to say how much the temperature influenced each test. The temperature elongation can be written as $\delta = \alpha \cdot L \cdot \Delta T$, where $\alpha = 1.2 \cdot 10^{-5}$ 1/K is the elongation coefficient for steel and concrete, $L$ is the length [m] and $\Delta T$ is the temperature change [K]. With a test length of $L$ = 42

mm and a temperature change of $\Delta T$ = 1 K, we obtain a theoretical elongation of $\delta = \alpha \cdot L \cdot \Delta T = 1.2 \cdot 10^{-5} \cdot 0.042 \cdot 1 = 0.0005$ mm.
This theoretical value can be compared to results from the temperature-trend-curve measurements, for e.g. test no. 20 and 34. For test no. 20 the deformation increased 0.00035 mm during 6000 seconds (1.7 hours), which corresponds to $5.8 \cdot 10^{-8}$ mm/sec. The change during the actual test was as mentioned higher. For test no. 34 the deformation increased 0.0006 mm during 6000 seconds (1.7 hours), which corresponds to $10 \cdot 10^{-8}$ mm/sec.
The deformation at the start of phase three, $\delta_{2-3}^{\mathrm{U}}$, varies from 0.0015 to 0.014 mm. A one degree change of the temperature may thus increase the deformation with as much as 33% of $\delta_{2-3}^{\mathrm{U}}$.

- **Creep**. The creep deformation can be written as $\varepsilon_{cr} = (\sigma/E) \cdot \varphi$, where $\sigma$ is the stress, $E$ is the modulus of elasticity and $\varphi$ is the creep factor. The mean stress during the fatigue test was $\sigma = F/A = 5350 \cdot 4/(\pi \cdot 74 \cdot 74)$ MPa = 1.24 MPa and the modulus of elasticity was $E$ = 32.2 GPa. The creep factor for indoor conditions can according to BBK94[6] be set to $\varphi = 3$. Due to drying of the specimen, the moisture conditions will change during the tests. However, $\varphi = 3$ will be used as a conservative estimation. This gives $\varepsilon_{cr} = (\sigma/E) \cdot \varphi = 1.24/32200 = 1.16 \cdot 10^{-4}$. If we assume that about 10 % of the total creep deformation takes place during the test, we will for a test length of $L$ = 42 mm have a creep deformation of $\delta_{cr} = 0.1 \cdot \varepsilon_{cr} \cdot L = 0.1 \cdot 1.16 \cdot 10^{-4} \cdot 42 = 0.00048$ mm. The creep deformation is thus of a smaller order of magnitude than the deformations at the start of phase three.

- **Shrinkage**. The total shrinkage deformation after long time for indoor conditions can with BBK94[6] be estimated to $\varepsilon_{shr} = 0.0004$. If we assume that about 10 % of the total shrinkage takes place during the test we will for a test length of $L$ = 42 mm have a shrinkage deformation of $\delta_{shr} = 0.1 \cdot \varepsilon_{shr} \cdot L = 0.1 \cdot 0.0004 \cdot 42 = 0.0017$ mm. This is of the same order of magnitude as the deformation at the start of phase three.

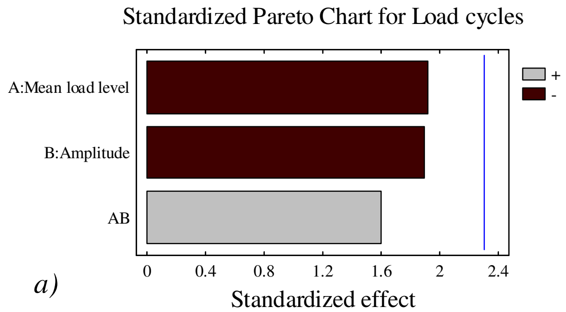
- **Static strength.** The highest influence on the number of load cycles to failure has probably the variation of the static tensile concrete strength between each specimen.

- **Scatter in test results**. The most influencing factor, at least theoretical, seems to be shrinkage. The influence of the shrinkage and temperature is different from test to test. For the tests that lasted longer than 600 000 load cycles (3 tests) the temperature influence is more crucial than for the tests that lasted for a shorter period of time (for these tests the temperature influence is probably almost negligible). This could also be said for the shrinkage. However, the biggest influence on the result in these tests is believed to be the variation of the static tensile concrete strength between each specimen.

## 4.2 Load level and amplitude

Which of the two varied factors, i.e. the load level and the amplitude, has the highest influence on the number of load cycles to failure? In Figure 9 the result from the analysis performed with factorial design is presented. The Pareto chart in Figure 9a shows that none of the two factors, neither the amplitude nor the load level, has any significant influence on the number of load cycles to failure – none of them reaches beyond the vertical line that represents a statistical significance at the 95 % confidence level. Nor is there any interaction for the two that has a significant influence on the result, see *AB*. One could say that they both have a somewhat equal influence on the number of load cycles. In Figure 9b, the main effects plot, it is shown that if the load level is low it results in a high number of load cycles which is not surprising. The same could be said for the amplitude, i.e. a low amplitude results in a high number of load cycles. In this context it must be remembered that if the amplitude is too low the test becomes a test with sustained load. In the interaction plot, Figure 9c, it is shown that no interaction is shown for the two. A high load level and a high amplitude give the lowest number of cycles to failure. It is also shown that a low load level and a high amplitude give approximately the same number of load cycles as if the load level is high and the amplitude is low.

With the help of the results from this analysis it is not possible to say which one of the two factors that has the highest influence on the number of load cycles to failure.




*a)*

*b)* *c)*

*Figure 9 - Results from analysis with factorial design in Statgraphics. a) A Pareto chart, b) the Main effects plot and c) the Interaction plot.*

Why this result then? As mentioned earlier the analysis assumes that the levels are fixed which they are not entirely. The reason for this is the variation in the static uniaxial tensile strength, which is the basis for the load levels. For example, when it is assumed that the load level is 60% of $F_{peak}$ it could as well be 50 % or 70 %. Another factor that can influence the result, even though the highest efforts have been made to reduce it, is the variation in temperature for the fatigue tests that lasted for a longer time (a few days or more). Since the specimens were not sealed during the fatigue tests, the moisture content has been changed which induces shrinkage. According to Möller et al.[11] the tensile strength drops when the drying process starts and with time the moisture gradient is equalised and after 1 or 2 months the tensile strength has reached its full capacity again. Another phenomenon that is connected to the nature of a fatigue test is the time which introduces creep effects.

A reason for the somewhat unclear result can be that the two chosen amplitudes and load levels are too close to each other. Perhaps a more distinct result would have been obtained if there had been a higher difference between the chosen levels. However, the method seems to be a suitable method to use in experiments since it can give additional bonus information. An example where the method has been used successfully is e.g. Utsi[12] where influencing factors on concrete mixtures' properties have been studied.

### 4.3 Deformation rate

From the results in Table 2 several interesting findings are worth comments. If the slopes, i.e. $\delta_{\alpha}^{\mathrm{U}}$ and $\delta_{\alpha}^{\mathrm{L}}$, for the upper and lower curves in the fatigue test are compared the upper curve is steeper (except for one test i.e. no. 32). In other words the two curves are separating which increases the deformation amplitude (compare $\delta_{1-2}^{\mathrm{A}}$ and $\delta_{2-3}^{\mathrm{A}}$). This deformation amplitude reaches its maximum value at the end of each test (see $\delta_{\max}^{\mathrm{A}}$). This should at first glance be more pronounced for the tests where the amplitude as well as the load level have been high since the specimen is more strained in these cases but the phenomenon could be found for all variations in load level and amplitude.

Another thing that can be found in Table 2 is that there is a big difference between the deformation rate ($\delta_{\alpha}^{\mathrm{U}}$) for the tests that lasted below about 3000 load cycles and the ones that lasted for more than 120 000 load cycles. With this in mind one can suspect a sort of fatigue limit in the sense that below a certain load level there is a need for many load cycles before failure occurs. Where this limit is, is not possible to say from the results in this investigation only that there is a very low deformation rate for a mean load level and an amplitude less than 40% of $f_{\mathrm{peak}}$.

In Figure 10 another interesting result is shown. In the figure the deformation rate for the upper fatigue curve for phase 2, $\delta_{\alpha}^{\mathrm{U}}$**,** is shown on the y-axis and the logarithm of the number of load cycles where the failure phase begins, $\log n_{2-3}^{\mathrm{U}}$, in the fatigue tests is shown on the x-axis. The interesting thing is that there is a very distinct difference between the tests that have lasted for more than approximately 300 load cycles (approximately log $n$ equal to 2.5) compared to the others, if the deformation rate is compared. The test either breaks almost directly or it lasts for very many load cycles. A regression analysis has been performed and the equation becomes:

$$\delta_{\alpha}^{\mathrm{U}} = 0.0165 \cdot e^{\left(-2.83 \cdot \log n_{2\text{-}3}^{\mathrm{U}}\right)} \qquad (1)$$

where $\delta_{\alpha}^{\mathrm{U}}$ is the deformation rate for the upper fatigue curve, [mm/load cycle].

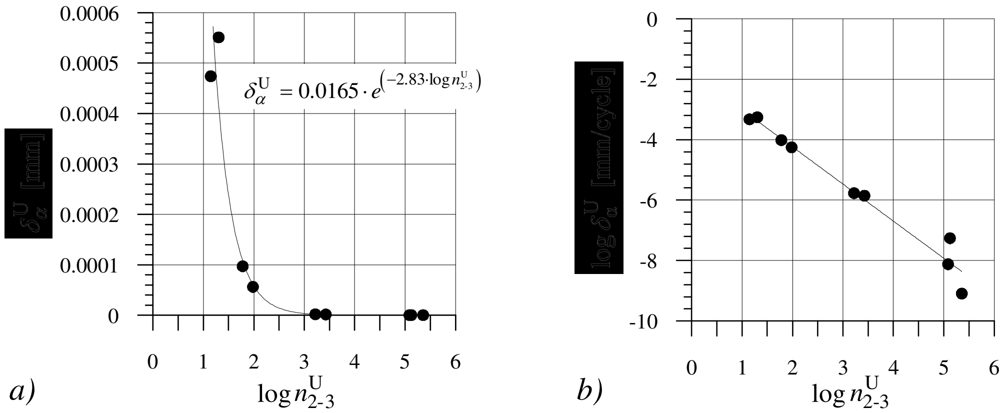



*Figure 10 – a) Deformation rate of phase 2, i.e. $\delta_\alpha^U$, on the y-axis versus the logarithm of the number of load cycles where phase 3 begins, $\log n_{2\text{-}3}^U$, on the x-axis. Regression analysis is based on y =exp(a+bx). The coefficient of determination, r-squared, is 0.96. b) Log-log curve.*

Using a log-log scale the curve can approximately be written as:

$$\log \delta_\alpha^U = -2 - 1.2 \cdot \log n_{2\text{-}3}^U \tag{2}$$

which seems to indicate a log-scale linear relationship between the deformation rate $\delta_\alpha^U$ and the number of load cycles $n_{2\text{-}3}^U$ when phase 3 begins.

## 4.4 Wöhler curve

The most common way to present results from fatigue tests is to use Wöhler curves, Wöhler[13] (1858-70). Over the years several Wöhler curves have been proposed by researchers regarding cyclic loading in compression, but not so many that regard cyclic loading in tension.

The results can be compared with an equation proposed by Tepfers[14] for cyclic splitting tension load:

$$S_{\max} = \frac{f_r^{\max}}{f_r^{'}} = 1 - \beta(1-R)\log N = 1 - \frac{1}{C}(1-R)\log N \tag{3}$$

Here $N$ is the number of load cycles up to fatigue failure, $R = f_r^{\min} / f_r^{\max}$, $f_r^{\max}$ is the upper limit of fluctuating splitting stress in tension, $f_r^{\min}$ is the lower limit of fluctuating splitting stress in tension and $f_r^{'}$ is the static splitting strength in tension. Tepfers performed his fatigue tests on 150 mm cubes with two different tensile strengths, approximate 3.4 MPa and 4 MPa (compressive concrete strengths of 40 MPa and 56 MPa). Tepfers performed the tests with three levels of $R$ = 0.2, 0.3 and 0.4 in combination with $S_{\max}$ = 0.7, 0.75, 0.80, 0.85, 0.90 and 0.95. The analysis of all 83 tests gave a mean value of the coefficient $\beta$ (=1/$C$) of 0.0597 (normal distribution, with a standard deviation $s$ of 0.0206). Since Tepfers assumed that uncertainties in the fatigue test could begin to appear for $S_{\max} \geq 0.80$ he chose to exclude these tests. The analysis based on the 12 remaining tests gave $\beta = 0.0675$ ($s = 0.0113$). However, since Tepfers & Kutti[15] in an earlier research project had recommended $\beta = 0.0685$ ($s = 0.0116$) for cyclic compressive load and that $\beta = 0.0685$ was within the confidence limits for the tension load tests, Tepfers believed it could also be used for fatigue subjected to tensile stresses.

Another model for tension is proposed in CEB-FIP[16] and can be written as:

$$S_{\max} = \frac{f^{\max}}{f^{\text{static}}} = 1 - \frac{1}{C}\log N = 1 - \frac{\log N}{12} = 1 - 0.083\log N \tag{4}$$

where $f^{\max}$ is the maximum tension stress and $f^{\text{static}}$ is the fatigue reference strength.

In Figure 11 Eqs. (3) and (4) are shown together with the test results for $R$ = 0.33 (normally a Wöhler curve is plotted for $R$ = constant ($\sigma_{\min}/\ \sigma_{\max}$), so therefore not all performed fatigue tests have been included). In the figure test no. 20 is also plotted, a so-called run-out, a test which was stopped at 5 million load cycles, since there were no signs of an imminent fatigue failure.

If the Eqs. (3), (4) and the test results are compared, the model proposed in CEB-FIP[16] is the most conservative. The equation by Tepfers i.e. Eq. (3) gives a somewhat longer fatigue life than the tests performed in this investigation. One explanation of this difference lies in the two different test methods that have been used, since the uniaxial tensile test is very sensitive to any cracks and defects in the notch area.

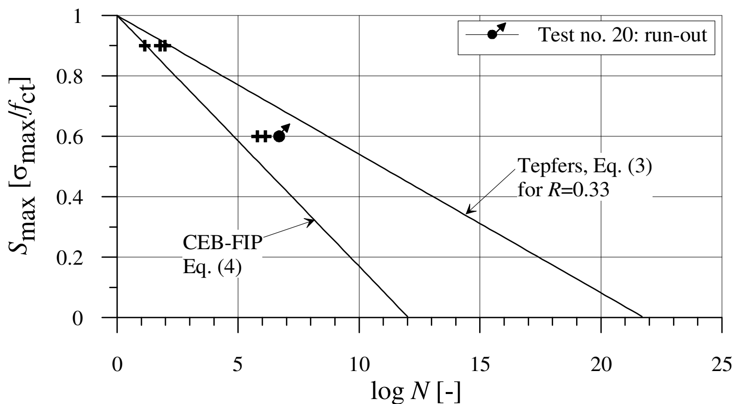



*Figure 11 - Wöhler curves for cyclic load in tension proposed by CEB-FIP[16] and Tepfers[14] (plotted with $\beta = 0.0685$, R = 0.33). The graph shows also test result for R = 0.33.*

In Figure 12 all the fatigue tests that have been performed are shown together with Eq. (3) for $R$ = 0, 0.14, 0.33 and 0.5. As can be seen some tests end to failure after fewer cycles than predicted by Eq. (3) and the scatter is considerable. Eq. (3) for $R$ = 0 gives almost the same number of cycles to failure as was observed in the tests with $R$ = 0.33.

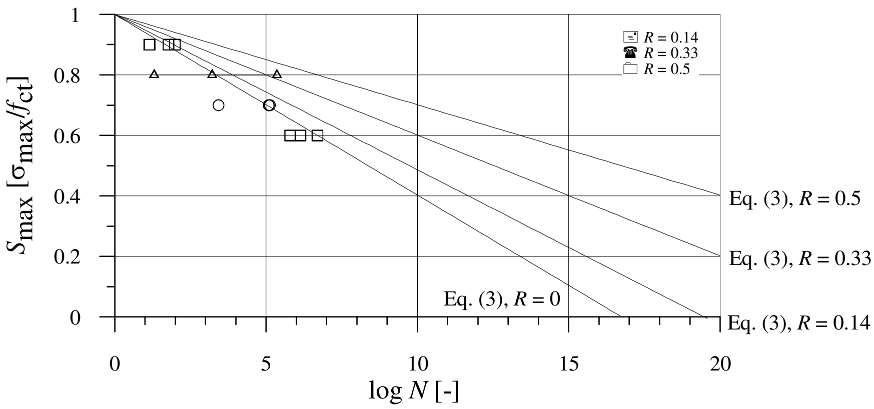



*Figure 12 – Figure showing results from all performed fatigue tests. The result is compared with an equation proposed by Tepfers[14].*

## 5 CONCLUSIONS

The findings in this investigation can be summarised as follows:

- The method used to design the experiments performed in this investigation, i.e. factorial design, did not give any evident result which of the two factors varied in the fatigue test that had the highest influence on the number of load cycles to failure (the load amplitude and the mean load level). Both of the factors seem to be of the same importance.

- The results show that there is a big difference between the deformation rate for the tests that lasted below about 3000 load cycles and the ones that lasted for more than 120000 load cycles. To obtain more than 1000 load cycles in the performed tests, the deformation rate must be less than 0.00005 mm/cycles. It appears that a certain fatigue limit exists below which a clearly greater number of load cycles is required for failure. From this research the exact limit cannot be predicted, but for tests with a mean load level of 40% of $f_{peak}$ and an amplitude of 40% of $f_{peak}$, a very low deformation rate has been obtained.

- The test results obtained for $R = \sigma_{min}/\sigma_{max} = 0.33$ is less conservative than the equation proposed by CEB-FIP[16] but more conservative than the equation proposed by Tepfers[14].

The scatter in the test results and the influence of temperature variations and shrinkage are described in more detail in Thun[9]. The scatter in the results from the performed fatigue tests, require that further tests are carried out in order to give safe methods to be used in assessment situations replacing the conservative methods in the present codes.

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