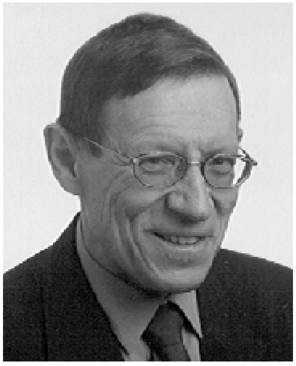
Partial Coefficient for Thermal Cracking Problems Determined by a Probabilistic Method

Martin Nilsson
Doctoral Student, Tech. Lic.
Department of Civil and Mining Engineering
Division of Structural Engineering
Luleå University of Technology
SE-971 87 Luleå, Sweden
E-mail: Martin.Nilsson@ce.luth.se



Lennart Elfgren
Professor, Tech. Dr.
Department of Civil and Mining Engineering
Division of Structural Engineering
Luleå University of Technology
SE-971 87 Luleå, Sweden
E-mail: Lennart.Elfgren@ce.luth.se

ABSTRACT

The aim of this work is to calculate partial coefficients for thermal cracking problems of young concrete and to compare the results with the values stated in the Swedish building code for bridges, [1]. The code values are only based on experiences and logical reasoning, whereas the calculated values form a more theoretical base for their determination. The coefficients are calculated with a probabilistic method. Various different possible variations of the used variables have been studied showing the wide range of possible results depending on the input. However, with use of material properties and reasonable assumptions related to thermal cracking problems, fairly good agreement has been found between the stated values in the Swedish code [1] and the values obtained through the probabilistic method.

The calculated values are based on many assumptions and assumed values and should therefor not be seen as what is right but rather more as an indication on the reasonableness of the values stated in the Swedish code. Further investigations, calculations and judgements should be performed before wider conclusions can be drawn.

Keywords: Partial Coefficients, Safety Factors, Young Concrete, Probabilistic method, Cracking

1 INTRODUCTION

A structure or a structural member should be designed in such a way that safety and serviceability are always maintained. This means that no relevant limit state conditions should be exceeded with an in beforehand determined probability. For young concrete structures it is important to prevent surface and through cracks due to e.g. temperature and/or temperature gradients during the hydration phase. Such cracks do not affect the total bearing capacity of a structure, the safety, but can influence the aesthetics and cause leakage and durability problems, the serviceability, and must be taken care of by e.g. injection.

The risk of thermal cracking in young concrete structures is commonly estimated as the ratio between the calculated maximum tensile stress and the actual tensile strength. Alternatively, the ratio between the calculated maximum tensile strain and the actual ultimate tensile strain is used, which will be the case here. If a determined ratio is smaller than a so-called crack safety value, a structure is assumed to fulfil the requirements of no thermal cracking. Depending on the effects of cracking and the accuracy in determining material properties, the Swedish building codes for bridges, [1], states different crack safety values as measures of the risk of cracking.

The risk of cracking due to temperature and temperature gradients can be estimated, according to [1], in three different methods. In Method 1 certain demands are specified on i.e. the casting and the air temperatures, the maximum cement content and the minimum value of the water cement ratio. Demands are also stated on the thickness and height of the structural members, the casting length, and when form stripping is allowed. In Method 2 and Method 3, which is more elaborate, certain values of the crack safety are prescribed depending on the accuracy in the determination of material data. Method 2 implies that requirements in a certain handbook, [2], should be applied. The requirements have been established by numerous thermal stress analyses. Further, material data that should be used are given in the code. In Method 3, the risk of cracking is estimated very accurately with tried and documented computer software and material properties.

The risk of cracking should not be larger than the crack safety values given in Table 1. The environmental classes referred according to [1] in the legend of the first column are according to the Swedish building code for concrete, [3]. Environmental class A2 stands for “Moderately reinforcement aggressive”, class A3 stands for “Very reinforcement aggressive” and class A4 stands for “Extremely reinforcement aggressive”, further see Section 4.2.

Table 1. Crack safety values for Method 2 and Method 3 given in [1]. For Method 2 values from the two right columns are used where C is the cement content [kg/m³]. Environmental class A2 stands for “Moderately reinforcement aggressive”, class A3 stands for “Very reinforcement aggressive” and class A4 stands for “Extremely reinforcement aggressive”.

Environm. class	Method 3 Complete material data	Method 2 Material data given in the code	
		$360 \leq C \leq 430$kg/m^3	$430 \leq C \leq 460$kg/m^3
A2	1.11	1.25	1.42
A3	1.18	1.33	1.54
A4	1.25	1.42	1.67

The crack safety values can be referred to what usually are called partial coefficients based on probabilistic methods, see e.g. [4], [5], [6], [7] and [8]. Determination of partial coefficients will be presented here. Further, a determination of partial coefficients, that is crack safety values, for thermal cracking problems will follow as an attempt to indicate the reasonableness in the values given in [1]. The method and the results are more thoroughly presented and described in [8]. The determination is based on material properties, assumption on load situations and other conditions typical for thermal cracking problems.

2 PARTIAL COEFFICIENTS

2.1 Limit state function and safety index

The safety against failure can be estimated by a limit state condition in terms of a resistance parameter r and a stress parameter s. The limit state condition, $\Theta(\cdot)$, can be expressed as the resistance parameter r reduced by the stress parameter s as

$$\Theta(\cdot) = r - s \geq 0 \tag{1}$$

Usually, the resistance parameter r is the material strength and the load parameter s is the stresses caused by acting loads. Depending on their relative size, the limit state condition is not exceeded if the resistance is larger than or equal to the stress, $r \geq s$, and it is exceeded if the resistance is smaller than the stress, $r < s$.

The two parameters are regarded as two normally distributed stochastic variables with given probability density functions, $f_r(r)$ and $f_s(s)$, see Figure 1a). From the presumption that the resistance parameter r and the stress parameter s are stochastic variables, the limit state condition is also a stochastic variable. Assuming the resistance parameter r and the stress parameter s being normally distributed also the limit state condition Θ is normally distributed with a probability density function $f_\Theta(\Theta)$, Figure 1b), where β is the so-called safety index.

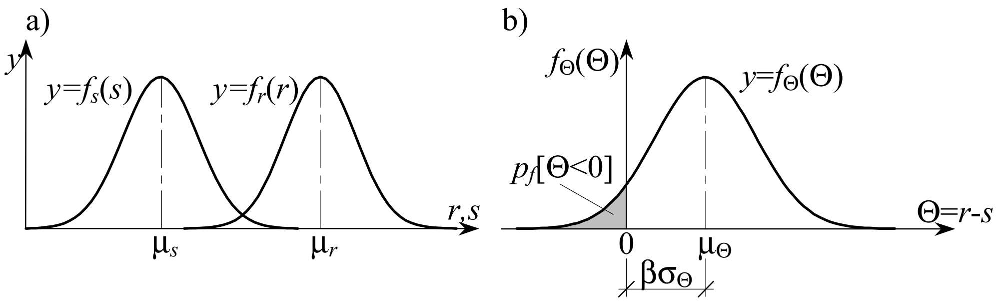


Figure 1. a) Probability density functions for the stress parameter, $f_s(s)$, and the resistance parameter, $f_r(r)$, b) Probability density function for the limit state condition Θ, $f_\Theta(\Theta)$.

The probability of exceeding a limit state condition, $p_f[\Theta = r\text{-}s < 0]$, is equal to the area of the shaded surface in Figure 1b). In the figure, the distance, with the standard deviation σ_Θ as unit, from the mean value μ_Θ to the failure limit, $\Theta = 0$, is written as $\beta\sigma_\Theta$. The coefficient β is the so-called safety index, introduced by Cornell in [9], and is, according to the figure, determined as

$$\beta = \frac{\mu_\Theta}{\sigma_\Theta} \tag{2}$$

How much larger the resistance r should be than the stress s is often specified in building codes in different safety classes and through specified values of the safety index β. The safety index β is defined by a formal probability of failure, that is, of exceeding the limit sate condition. The safety index β is often coupled to safety classes in building codes, see e.g. [6], [7], [10]. If the risk of human injuries is low, often referred to safety class 1, the probability of failure is $p_f = 10^{-4}$ and the safety index $\beta = 3.72$. The same principle applies to safety classes 2 and 3, see Table 2.

Table 2. Correspondence between safety class, safety index and probability of failure, [1], [10].

Safety class	1	2	3
Safety index β Probability of failure, p_f	3.72 10^{-4}	4.26 10^{-5}	4.75 10^{-6}

2.2 Partial coefficients

The partial coefficient method is based on characteristic values and partial coefficients for verification that prescribed safety requirements are fulfilled. Generally, for the limit state condition in Eq. (1), partial coefficients are used as follows

$$\Theta = r_d - s_d = \frac{r_c}{\gamma_r} - \gamma_s s_c \geq 0 \tag{3}$$

where d indicates design values, c indicates characteristic values and γ_r and γ_s are the partial coefficients for the resistance parameter r and the stress parameter s, respectively.

For the risk of thermal cracking of young concrete, the crack safety values in Table 1 are the product of the partial coefficients for the resistance parameter r and the stress parameter s, $\gamma_r\gamma_s$, according to, compare with Eq. (3),

$$\frac{r_c}{s_c} \geq \gamma_r \gamma_s \tag{4}$$

In this case, all partial coefficients have been collected in one coefficient limiting the ratio between the resistance parameter and the load parameter.

3 THE PROBABILISTIC METHOD

3.1 Equations for determination of partial coefficients

A method further referred to as the probabilistic method will be used to determine alternative values of the partial coefficients, safety values, for thermal cracking problems, given in Table 1. The method has the advantage of being consequent but it also includes many approximations.

The results can therefore not be used directly without additional judgements. The following determination of the partial coefficients will be formulated in terms of strains. The procedure in general is based on a method presented by Lars Östlund in [11], reprinted in [12], and adopted on thermal cracking problems in [8]. As design condition with partial coefficients for thermal cracking problems, Eq. (4) will be used as the limit state condition.

3.1.1 Resistance parameter

The resistance parameter r is expressed as, [11]

$$r = C_r a \rho \varepsilon \tag{5}$$

where C_r is a factor describing uncertainties in the calculation method on the resistance parameter such as determination of material properties. C_r is a stochastic variable with mean μ_{Cr} and coefficient of variation V_{Cr}. a is a geometric quantity (eg cross-section area). a is a stochastic variable with mean μ_a and coefficient of variation V_a. ρ is a factor transferring concrete strain from test specimen at failure to concrete strain in real structures. ρ is a stochastic variable with mean μ_ρ and coefficient of variation V_ρ. ε is the actual concrete ultimate strain. ε is a stochastic variable with mean μ_ε and coefficient of variation V_ε. The stochastic variables r, C_r, a, ρ and ε are assumed to be logarithmic normally distributed.

The mean value of the resistance parameter is

$$\mu_r = \mu_{Cr}\mu_a\mu_\rho\mu_\varepsilon \tag{6}$$

and the coefficient of variation, if terms of higher order are neglected,

$$V_r \approx \sqrt{V_{Cr}^2 + V_a^2 + V_\rho^2 + V_\varepsilon^2} \tag{7}$$

Eq. (5) divided by Eq. (6) gives, if using characteristic values,

$$\frac{r_c}{\mu_r} = \frac{C_{rc}}{\mu_{Cr}} \frac{a_c}{\mu_a} \frac{\rho_c}{\mu_\rho} \frac{\varepsilon_c}{\mu_\varepsilon} \tag{8}$$

which will be used further on in the final calculation of the partial coefficients, see Eq. (25) below.

3.1.2 Load parameter

The load parameter s for thermal cracking problems can be formulated, in terms of strains, as

$$s = C_s \gamma_R (b(\varepsilon_{T1} + \varepsilon_{T2}) + c\varepsilon_{sh})$$

where C_s is uncertainties in the calculation method on the load parameter and is assumed to have the same value for all the loads. C_s describes uncertainties in the determination of the strains by e.g. manual methods, see [13] and [14], or by finite element calculations, see [15]. C_s is a stochastic variable with mean μ_{Cs} and coefficient of variation V_{Cs}. γ_R is the coefficient of restraint and is a deterministic coefficient, $0 \le \gamma_R \le 1$. For further explanations and the determination of the coefficient of restraint, see [8]. ε_{T1} is the non-elastic strain of volume changes from differences between the casting temperature and the adjacent temperature. ε_{T2} is the non-elastic strain of volume changes from differences between the maximum temperature and the casting temperature. Below, the temperature-induced strains are combined into one parameter, ε_T, which is a stochastic variable with mean μ_T and coefficient of variation V_T. ε_{sh} is the strain of volume changes from shrinkage and is a stochastic variable with mean μ_{sh} and coefficient of variation V_{sh}. b and c are both deterministic coefficients, $0 \le b$ and $0 \le c$. The stochastic variables ε_T and ε_{sh} are assumed to be normally distributed. The deterministic coefficients b and c are used when either the temperature induced strain is of greater importance than the shrinkage strain, or the opposite. Now, the load parameter is

$$s = C_s \gamma_R (b\varepsilon_T + c\varepsilon_{sh}) \tag{9}$$

The variables are put together so that the mean value of the stress parameter is

$$\mu_s = \gamma_R \left(b\mu_T + c\mu_{sh} \right) \tag{10}$$

By introducing the following relation

$$\frac{C_r}{C_s} = C; \quad V_C = \sqrt{V_{Cr}^2 + V_{Cs}^2} \tag{11}$$

the limit state condition is simplified to

$$\Theta(\cdot) = Ca\rho\varepsilon - \gamma_R (b\varepsilon_T + c\varepsilon_{sh}) \tag{12}$$

In the calculation of the partial coefficients for thermal cracking problems of concrete, it is very difficult to give any absolute values of the mean values of the strains of shrinkage and temperature changes. However, the relation between them is easier to estimate. Therefore, a coefficient ν_{sh} is introduced stating the ratio between the mean values of the strains of shrinkage and of the temperature change

$$\nu_{sh} = \frac{c\mu_{sh}}{b\mu_T} \tag{13}$$

3.1.3 Design condition

When calculating partial coefficients by the probabilistic method, the following design values and help values κ are used for the stochastic variables r, ε_T and ε_{sh}.

$$r_d = \mu_r \exp(-\alpha_r \beta V_r) \quad \kappa_r = r_d V_r \tag{14}$$

$$\varepsilon_{T,d} = \mu_T \left(1 - \alpha_T \beta V_T\right) \quad \kappa_T = -b\gamma_R \mu_T V_T \tag{15}$$

$$\varepsilon_{sh,d} = \mu_{sh} \left(1 - \alpha_{sh} \beta V_{sh}\right) \quad \kappa_{sh} = -c\gamma_R \mu_{sh} V_{sh} \tag{16}$$

When using design values in Eq. (3), the equal sign is valid, which together with Eq. (9) gives

$$r_d - b\gamma_R \varepsilon_{T,d} - c\gamma_R \varepsilon_{sh,d} = 0 \tag{17}$$

In the expressions above, α are so-called sensitivity coefficients determined as

$$\alpha_i = \frac{\kappa_i}{\sqrt{\Sigma\kappa_i^2}} = \frac{\kappa_i}{\sqrt{\kappa_r^2 + \kappa_T^2 + \kappa_{sh}^2}}; \quad \text{with } i = r,\ T \text{ and } sh \tag{18}$$

and that must fulfil the condition

$$\alpha_r^2 + \alpha_T^2 + \alpha_{sh}^2 = 1 \tag{19}$$

The sensitivity coefficients take values between -1 and 1 and are positive for favourable factors, the resistance parameters, and negative for unfavourable, the load/stress parameters. The larger the coefficient is, the larger the importance of the uncertainty is in the corresponding variable.

$c\mu_{sh} = \nu_{sh} b\mu_T$ according to Eq. (13) and design values according to Eqs. (14) to (16) inserted in Eq. (17) give

$$\frac{\mu_r}{b\gamma_R\mu_T}\exp(-\alpha_r\beta V_r) - (1 - \alpha_T\beta V_T) - \nu_{sh}(1 - \alpha_{sh}\beta V_{sh}) = 0 \tag{20}$$

By introducing the help variables

$$Z = \frac{\mu_r}{b\gamma_R\mu_T}$$

and

$$\psi_1 = (1 - \alpha_T\beta V_T) + \nu_{sh}(1 - \alpha_{sh}\beta V_{sh}) \tag{21}$$

Eq. (20) is simplified to

$$Z\exp(-\alpha_r\beta V_r) - \psi_1 = 0$$

where from

$$Z = \psi_1 \exp(\alpha_r\beta V_r) \tag{22}$$

Z can be determined if the values of α_i (with $i = r$, T and sh), β, ν_{sh}, b, c and V_i are known. The steps for calculating Z can be as follows:

(1) A value of α'_{sh} is assumed

(2) $\alpha'_T = \dfrac{\kappa_T}{\sqrt{\Sigma\kappa_i^2}} = \dfrac{-b\gamma_R\mu_T V_T}{-c\gamma_R\mu_{sh}V_{sh}}\alpha'_{sh} = \dfrac{V_T\alpha'_{sh}}{\nu_{sh}V_{sh}}$ is calculated

(3) ψ is calculated with Eq. (21), α'_{sh} and α'_T

(4) $r_d = \mu_r \exp(-\alpha_r\beta V_r) = \mu_r \dfrac{\psi_1}{Z} = b\gamma_R\mu_T\psi_1$ and $\kappa_r = r_d V_r$ are calculated

(5) $N = \dfrac{\sqrt{\Sigma\kappa_i^2}}{b\gamma_R\mu_T} = \sqrt{(V_T)^2 + (\nu_{sh}V_{sh})^2 + (\psi_1 V_r)^2}$

(6) $\alpha_{sh} = \frac{\kappa_{sh}}{\sqrt{\Sigma\kappa_i^2}} = \frac{-\gamma_R b\mu_T \nu_{sh} V_{sh}}{\sqrt{\Sigma\kappa_i^2}} = \frac{-\nu_{sh} V_{sh}}{N}$ is calculated and compared to α'_{sh}

(7) When $\alpha'_{sh} \approx \alpha_{sh}$, $\alpha_T = \frac{-V_T}{N}$ and $\alpha_r = \frac{\psi_1 V_r}{N}$ are calculated

(8) Check of $\Sigma\alpha_i^2 = 1$

(9) Z is calculated by Eq. (22).

The value of Z is used below in the calculation of the partial coefficients.

3.1.4 Partial coefficients

The design values in Eqs. (14) through (16) can alternatively be expressed with partial coefficients as

$$r_d = \frac{r_c}{\gamma_r} = \frac{\mu_r}{\gamma_r}\exp(-k_r V_r) \tag{23}$$

$$s_d = \gamma_s \gamma_R \left(b\varepsilon_{T,c} + c\varepsilon_{sh,c}\right) = \gamma_s \gamma_R \left(b\mu_T(1+k_T V_T) + c\mu_{sh}(1+k_{sh}V_{sh})\right) \tag{24}$$

which in the limit state condition, Eq. (3), give

$$\frac{\mu_r}{\gamma_r}\exp(-k_r V_r) - \gamma_s \gamma_R \left(b\mu_T(1+k_T V_T) + c\mu_{sh}(1+k_{sh}V_{sh})\right) \geq 0$$

With $Z = \mu_r/b\gamma_R\mu_T$, $\nu_{sh} = c\mu_{sh}/b\mu_T$ and $\psi_2 = (1+k_T V_T)+\nu_{sh}(1+k_{sh}V_{sh})$ it can be re-written as

$$\gamma_s \gamma_r \leq \frac{Z}{\psi_2}\exp(-k_r V_r) = \frac{Z}{\psi_2}\frac{r_c}{\mu_r} \tag{25}$$

giving the partial coefficients $\gamma_r\gamma_s$. Z is calculated according to Section 3.1.3 and r_c/μ_r is calculated from Eq. (8) with $x_{i,c}/\mu_i = \exp(-\alpha_i\beta V_i) = \exp(-k_i V_i)$. k_i depends on actual fractile value.

3.2 Numerical values

Calculations of partial coefficients for thermal cracking problems of young concrete have been performed by varying the variables shown in Table 3 and keeping all others constant.

ν_{sh} are defined by to Eq. (13) and states the ratio between the mean values of the strains of shrinkage and of the strains of temperature change. b and c are varied to simulate situations when one of the two strain components has smaller or larger influence. Especially in high strength concrete the shrinkage is considerable implying larger values of c. V_ε is the coefficient of variation of the actual concrete (actual ultimate strain ε_{cu}). V_C is the coefficient of variation of the methods used for estimating the risk of thermal cracking. Compare V_C with Methods 1 to 3 in Section 1 where e.g. V_C = 0.15 for Method 1, V_C = 0.10 for Method 2 and V_C = 0.05 for Method 3. These values are just an attempt to estimate the accuracy in the methods and should not be seen as what is right. The safety index β is varied to coincide with safety classes 1 and 3 with probabilities of failure of 10^{-4} and 10^{-6}, see Table 2.

Table 3. Variables varied in the determination of partial coefficients for thermal cracking problems.

Variable	Values				
ν_{sh}	$0.01\frac{c}{b}$	$0.20\frac{c}{b}$	$0.50\frac{c}{b}$	$1.00\frac{c}{b}$	$2.00\frac{c}{b}$
b	1/3	1	3		
c	1/3	1	3		
V_ε	0.05	0.10	0.15	0.20	0.25
V_C	0.05	0.10	0.15	0.20	0.25
β	3.72	4.75			

The coefficient of variation of the temperature induced strains is given the value $V_T = 0.08$ according to [16]. The coefficient of variation of the shrinkage is given the value $V_{sh} = 0.20$. This value is a bit smaller than what can be determined from [17]. The values of k_T and k_{sh} are 1.65 coinciding with 95 % fractile values of the temperature and shrinkage induced strains, respectively, see Table 4.

The coefficients of variations of the geometry parameter V_a and of the factor transferring strength in test specimens and in real structures V_ρ are both given the value 0, that is $V_a = 0$ and $V_\rho = 0$. The coefficient of variation of the geometry is assumed to be very low since in civil engineering structures, any divergences from the right measures do not affect the risk of thermal cracking. For the concrete ultimate strain, $k_\varepsilon = 0.13$, is chosen assuming a 45% fractile value. The high value of the ultimate strain for the concrete is chosen bearing in mind that thermal cracking only causes flaws and costs for repair and reduction of the life of the structure but not total failure. For the accuracy in design method, C, for the geometry parameter, a, and for the factor transferring the ultimate strain in test specimens and in real structures ρ, the coefficient k is chosen $k_C = k_a = k_\rho = 1.65$ assuming 5% fractile values, see Table 4.

Table 4. Constant values for the resistance parameters C, a, ρ and ε and the load parameters T and sh used in the determination of the partial coefficients.

k_C	V_a	k_a	V_ρ	k_ρ	k_ε	V_T	k_T	V_{sh}	k_{sh}
1.65	0	1.65	0	1.65	0.13	0.08	1.65	0.20	1.65

3.3 Calculation of partial coefficients

The following presumptions and values are used to illustrate the calculation of partial coefficients. Let the influence of the imposed volume changes be equal, $b = c = 1$. The mean value of the volume change due to shrinkage is one hundredth of the mean value of the imposed volume change due to the temperature change, $\nu_{sh} = 0.01 \cdot 1/1 = 0.01$. Further, the variation coefficients of the strength of the concrete and the calculation method are assumed to be five percent, $V_\varepsilon =$

V_C = 0.05. The safety index β = 3.72 corresponding to safety class 1 and corresponding to a probability of exceeding the limit state condition $p_f = 10^{-4}$. The following values for the resistance parameter, the sensitivity values α and the help values ψ, N and Z are obtained, Table 5 and Table 6.

Table 5. Calculated values for the resistance parameter.

V_r	C_c/μ_C	a_c/μ_a	ρ_c/μ_ρ	$\varepsilon_c/\mu_\varepsilon$	r_c/μ_r
0.071	0.921	1.000	1.000	0.994	0.915

Table 6. Calculated sensitivity values α and help-values ψ_1, N and Z.

α'_{sh}	α_T	ψ_1	N	α_φ	α_T	α_r	Z
-0.017	-0.682	1.213	0.117	-0.017	-0.682	0.731	1.470

The partial coefficient for this case is then calculated as, Eq. (25)

$$\gamma_r\gamma_s = \frac{Z}{\psi_2}\frac{r_c}{\mu_r} = \frac{1.470}{(1+1.65\cdot 0.08)+0.01(1+1.65\cdot 0.20)}0.915 = 1.174 \tag{26}$$

implying that the resistance parameter must be about 1.174 times larger than the load parameter for not exceeding the limit state condition.

All the partial coefficients calculated with values according to the description and Table 3 above are presented in Figure 2 to Figure 6 below. In all the diagrams, the curves from the lowest to the upper most represent V_C = 0.05, 0.10, 0.15, 0.20 and 0.25, respectively. See [8] for more descriptions of the calculations and the results.

In Figure 2 to Figure 6 it can be seen that with increased safety index β, the partial coefficient $\gamma_r\gamma_s$ increases and is varying over a larger range depending on the values of V_C. When the coefficient b increases also the partial coefficient increases, and when b decreases the partial coefficient decreases, compare Figure 3 and Figure 4 with Figure 2. For the coefficient c, the opposite is valid. When c increases, the partial coefficient decreases and when c decreases, the partial coefficient increases, compare Figure 5 and Figure 6 with Figure 2.

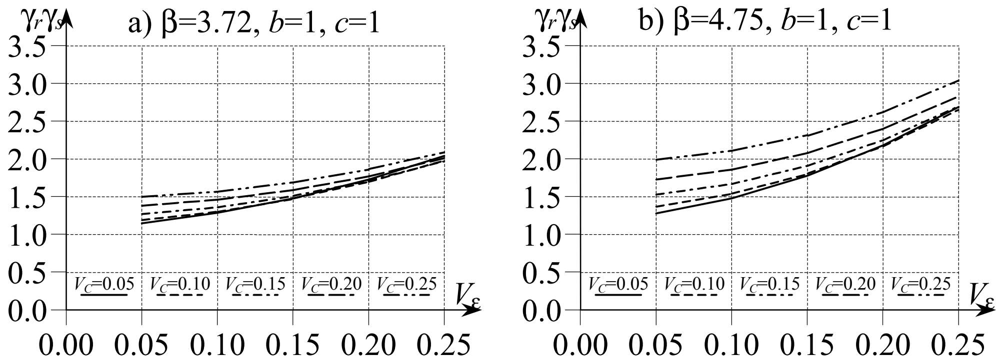


Figure 2. Partial coefficient $\gamma_r\gamma_s$ for a) β=3.72, b=1 and c=1, b) β=4.75, b=1 and c=1.

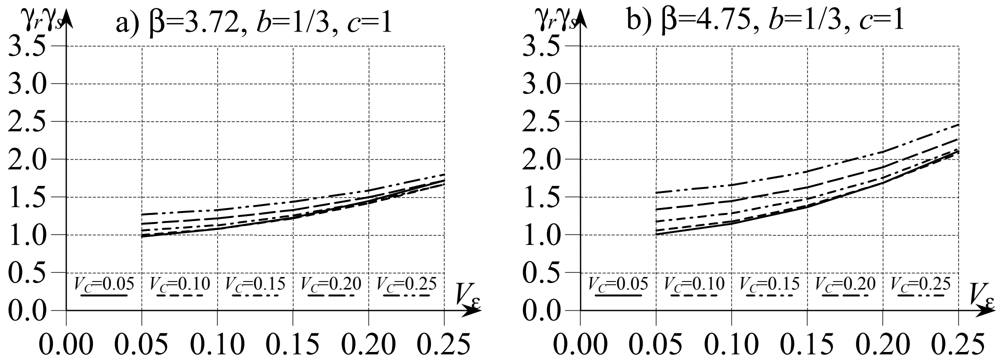


Figure 3. Partial coefficient $\gamma_r\gamma_s$ for a) β=3.72, b=1/3 and c=1, b) β=4.75, b=1/3 and c=1.

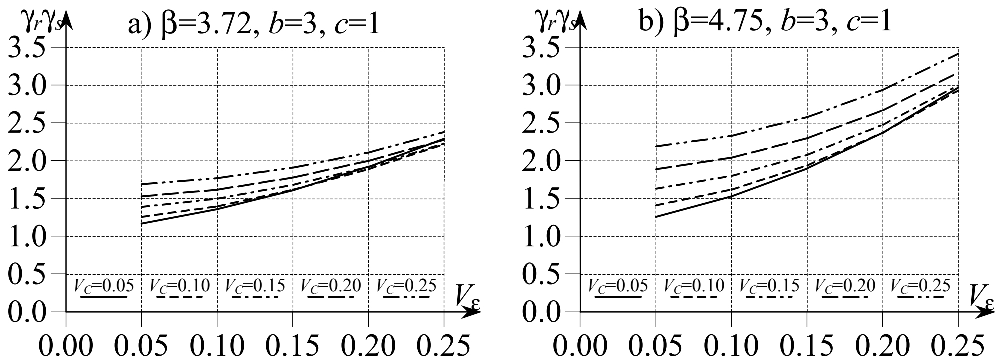


Figure 4. Partial coefficient $\gamma_r\gamma_s$ for a) β=3.72, b=3 and c=1, b) β=4.75, b=3 and c=1.

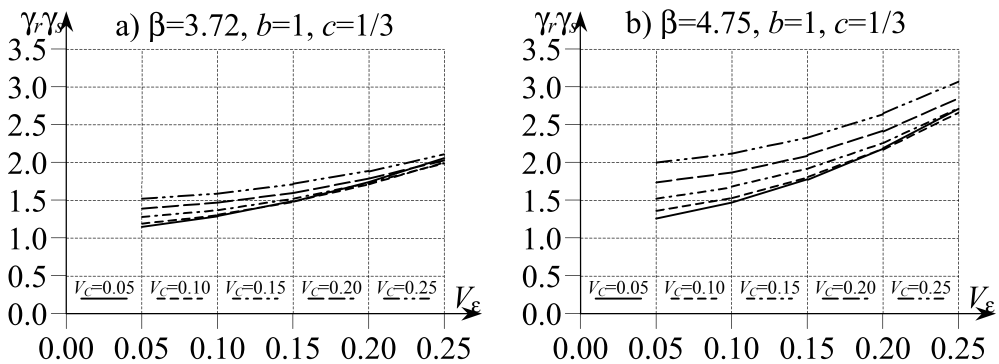


Figure 5. Partial coefficient $\gamma_r\gamma_s$ for a) β=3.72, b=1 and c=1/3, b) β=4.75, b=1 and c=1/3.

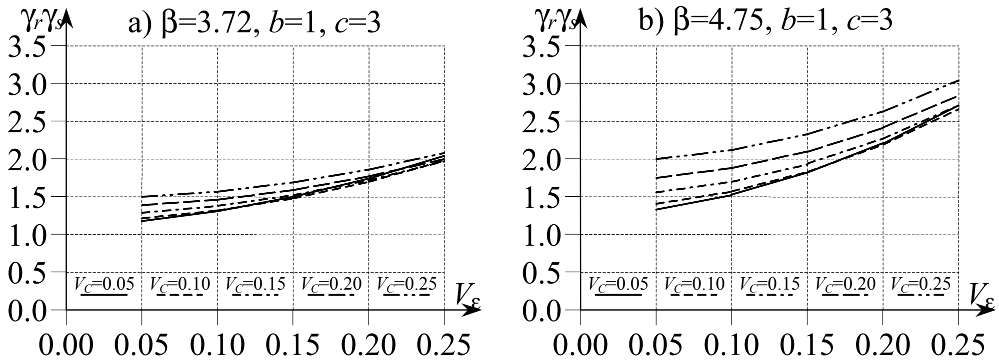


Figure 6. Partial coefficient $\gamma_r\gamma_s$ for a) β=3.72, b=1 and c=3, b) β=4.75, b=1 and c=3.

4 RESULTS

4.1 Final values of partial coefficients

Final values of the partial coefficient $\gamma_r\gamma_s$ are determined from the previous calculations with $b = c = 1$, $\beta = 3.72$ (probability of failure, $p_f = 10^{-4}$) and with coefficients of variation, $V_C = 0.05$ and $V_\varepsilon = 0.05$, 0.10 and 0.15. The values are chosen to coincide with the first row in Table 1. That is, for Method 3 (the column of complete material data) the models of analysis (computer software) are very well documented and tried and should give results not varying much from reality. Therefore, the coefficient of variation for the method of calculation is chosen to be small, $V_C = 0.05$. For Method 2, (columns for material data given in [1]) lots of calculations and judgements are behind, [2], implying good accuracy of the analyses, again $V_C = 0.05$. The differences in accuracy of material data are taken into account by varying the coefficient of variation of the material V_ε as stated, $V_\varepsilon = 0.05$, 0.10 and 0.15. Again, $k_T = k_{sh} = 1.65$ for 95 % fractile values. Further, as an extension of the final determination of the partial coefficients, 55 % fractile values are assumed for the temperature and the shrinkage induced strains to coincide with the assumed fractile value of the ultimate strain (45 % fractile), see Section 3.2. For environmental class A2 and $V_\varepsilon = 0.05$, 0.10 and 0.15, the partial coefficient $\gamma_r\gamma_s$ is taken as the values of the lowest curve in Figure 2a) presented in Table 7.

Table 7. Partial coefficient $\gamma_r\gamma_s$ from calculation with the probabilistic method for environmental class A2 and V_ε = 0.05, 0.10 and 0.15.

Environm. class	k_T, k_{sh}	Complete material data V_ε=0.05	Material data given in the code 360≤C≤430kg/m^3 V_ε=0.10	430≤C≤460kg/m^3 V_ε=0.15
A2	0.13 (55% fractile)	1.36	1.52	1.75
	1.65 (95% fractile)	1.15	1.29	1.48

4.2 Effects of exceeding the limit state condition

The calculation of partial coefficients above is chosen to be valid for environmental class A2. The effects of exceeding the limit state condition (cracking) in a structural member are smaller in environmental class A2 than in classes A3 and A4. Therefor an extra partial coefficient γ_n is introduced. The values of the extra partial coefficient γ_n are chosen as the mean ratio between the values in the rows in Table 1, see Table 8.

Table 8. Partial coefficient γ_n depending on environmental classes.

	Environmental class		
	A2	A3	A4
γ_n	1.00	1.07	1.14

Final values of the partial coefficient $\gamma_r\gamma_s$ are obtained from Table 7 with partial coefficient γ_n in Table 8, see Table 9.

Table 9. Final values of partial coefficient $\gamma_r\gamma_s$ as determined by probabilistic method.

Environm. class	k_T, k_{sh}	Complete material data	Material data given in the code	
			$360 \leq C \leq 430$kg/m^3	$430 \leq C \leq 460$kg/m^3
A2	0.13 (55% fractile)	1.15	1.29	1.48
	1.65 (95% fractile)	1.36	1.52	1.75
A3	0.13 (55% fractile)	1.23	1.38	1.58
	1.65 (95% fractile)	1.45	1.62	1.87
A4	0.13 (55% fractile)	1.32	1.48	1.70
	1.65 (95% fractile)	1.56	1.74	2.00

A comparison with the values that is stated in [1] and the values of the partial coefficients obtained by the probabilistic method are depicted in Figure 7. As can be seen, the values for $k_T = k_{sh} = 1.65$ (95 % fractile values) are somewhat higher than the values given in [1]. The values show good agreement even though the uncertainties in the chosen values of the variables used in the probabilistic method and that the partial coefficients stated in [1] only are based on experiences. For $k_T = k_{sh} = 0.13$ (55 % fractile values), the partial coefficients are much higher than the values in [1]. The reason for this is that with only 55 % fractile values of the temperature and the shrinkage induced strains, the risk of exceeding these values is increased. This implies an increased risk of exceeding the limit state condition, whereupon higher partial coefficients are needed.

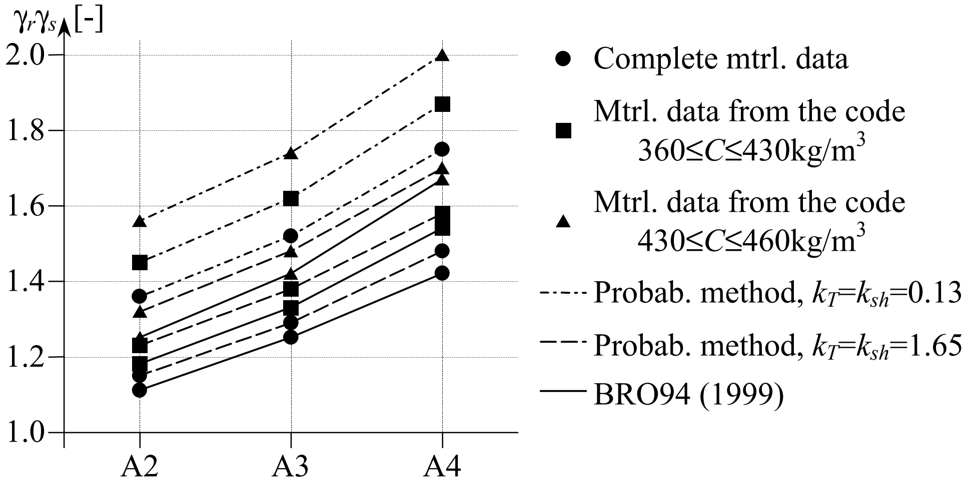


Figure 7. Comparison between partial coefficients stated in [1] and partial coefficients obtained by the probabilistic method.

5 DISCUSSION

It is possible to calculate partial coefficients for thermal cracking problems of young concrete. The values presented above coincide well with the crack safety values stated in the Swedish building code for bridges, [1]. However, the calculated values of the partial coefficient are based on many assumptions and simplifications and they shall not be seen as what is absolutely true right, further judgements are always necessary.

The used coefficients of variation of the thermal changes and of the shrinkage need further investigation. The values are roughly taken from [16] and are only assumed values that have not been well verified.

The crack safety values in [1] are all based on experience, so also these values are a bit vague. The calculated partial coefficients presented here can be seen as an attempt to verify the values in [1]. However, all estimations of the risks of thermal cracking of young concrete have to be based on more judgements and analyses of the problems as a whole rather than on the crack safety values given in [1].

The differences in the partial coefficient between the environmental classes need further investigations. The values that are stated in [1] are only based on logical arguments by the persons who have written the code, meaning that higher environmental class needs higher partial coefficients.

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