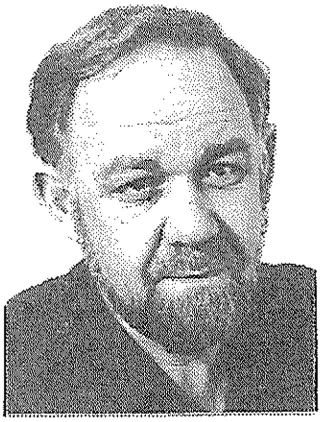

TESTS ON RECTANGULAR CONCRETE SLABS WITH HORIZONTAL RESTRAINTS ON THREE SIDES ONLY

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SYNOPSIS

A concrete slab, which is restrained from horizontal movement at the edges, obtains due to compression membrane forces or dome action a bearing capacity considerably larger than the load P_J predicted by the Johansen yield line theory.

In another paper /1/ the Authors examined the results of a number of tests carried out by the Authors and others. This led to the following estimate for the maximum load on a rectangular concrete slab with horizontal restraints on all four sides

$$P = P_J + h^2 f_{cyl}$$

where P_J is the Johansen load, h the slab thickness and f_{cyl} the concrete cylinder strength. This paper describes similar tests performed by the Authors and others on rectangular concrete slabs with horizontal restraints on only three sides. In this case

$$P = P_J + 0.7\ h^2 f_{cyl}$$

is found to be a good estimate for the bearing capacity.

Key words: Concrete slabs. Membrane action. Dome effect. Horizontal restraint.

1. MODEL TESTS

In the Structural Laboratory of DIAB six loading tests were carried out on concrete slabs with three sides clamped and horizontally restrained and with the fourth side simply supported.

1.1 The slab models

The six slabs were square with side lengths $l = b = 1250$ mm and a thickness h of about 40 mm. The concrete was made from rapid hardening cement and aggregates of sea materials. The concrete strength and modulus of elasticity given in Table 1 are mean results of 3 tests on 150×300 mm cylinders.

Three of the slabs were unreinforced. The other three slabs were reinforced by a galvanized square net of 1.8 mm bars per 25 mm. In Table 1 are given the steel areas per unit length, A_s in the bottom of the slab and A_s' in the top over the clamped supports, as well as the distances d and d' from the centroids of the steel areas to the opposite slab surfaces.

TABLE 1. Data for slab models.
Yield stress of reinforcing steel 330 MPa

Slab no.	h mm	A_s $\frac{mm^2}{m}$	A_s' $\frac{mm^2}{m}$	d mm	d' mm	f_{cyl} MPa	E_c MPa
13	43	102	102	35	29	20.9	18200
14	40	102	102	31	32	16.1	21400
15	36	102	102	28	29	17.5	17200
16	42	0	0			17.8	19600
17	40	0	0			15.3	18600
18	39	0	0			18.6	20200

1.2 The method of testing

The horizontal restraints consisted of a 625 mm wide concrete edge zone along three sides of the 1250×1250 mm slab model. The edge zone had the same thickness as the slab model and was reinforced as shown in Fig.1. On the fourth side of the slab model the protruding edge zones were connected by a varying number of 10 mm deformed bars. There were 4 connecting bars in slabs no.13 and 16, 2 in slabs no.14 and 17 and none in slabs no.15 and 18.

The vertical supports consisted of two rectangular steel frames. The slab rested on the frame called "support frame" in Fig.2, and the hogging bending moments in the slab over the supports were balanced by the frame marked "stability frame".

The loading was applied at 16 points situated 4 by 4 at 250 mm centre to centre, see Fig.2. The 16 point loads were interconnected by a system of yokes ending with a tension cell and a hydraulic jack. The initial load, consisting of the self-weight of the slab model and the loading arrangement, was 2.8 kN.

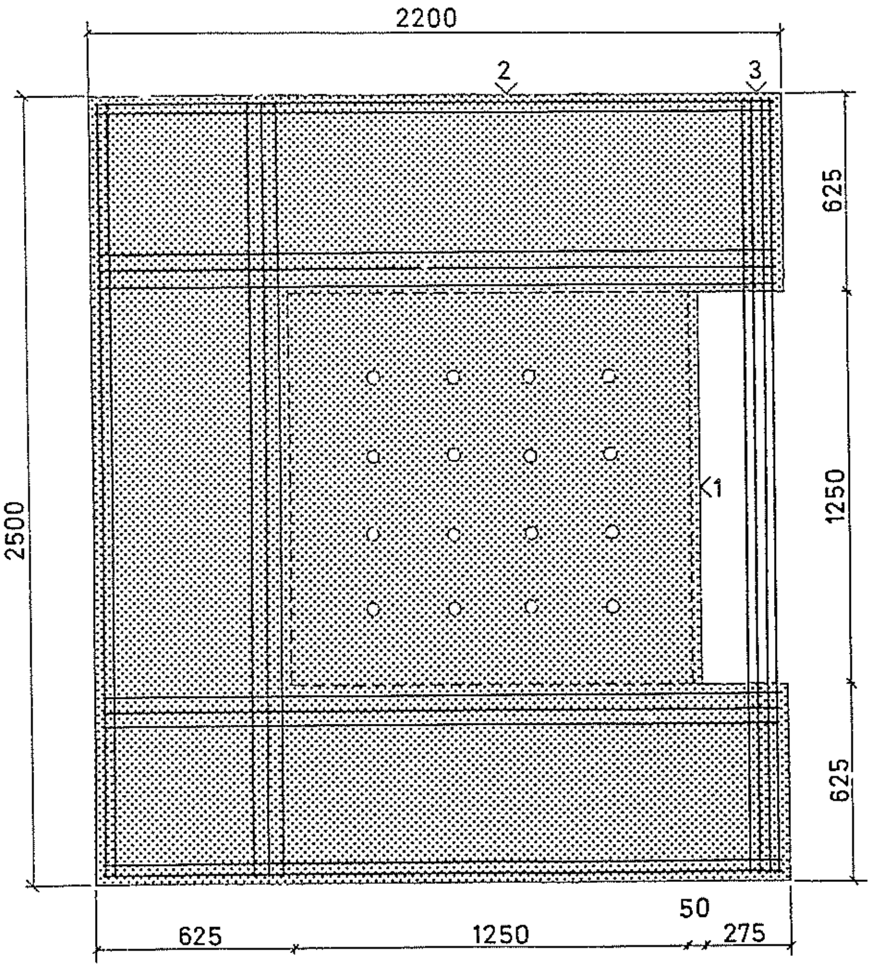

FIG. 1. Plan of slab model and edge zone.

The reinforcement shown is 10 mm bars. The number of connecting bars varied. Measurements in mm.

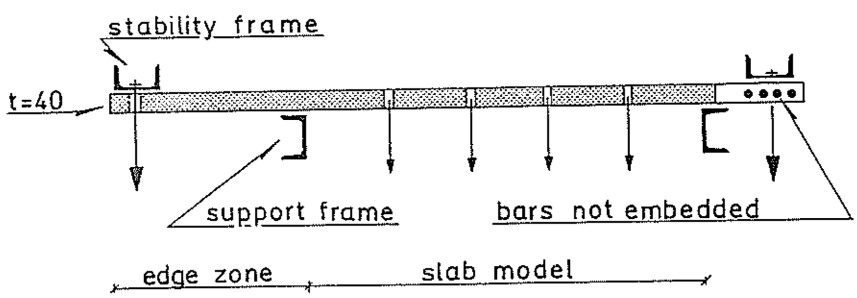

FIG. 2. Slab with supports and loading.

1.3 The test results

The load was applied at a rate of about 1 kN per minute in the beginning of each test and later at a constant rate of deflection of about 0.4 mm per minute.

During the tests vertical and horizontal displacements and the strain in the connecting bars were measured. Some of the results are shown in Table 2 and Fig. 3. The values given for u_2 and u_3 are the mean of measurements taken on opposite sides. The total force in the connecting bars is calculated from the measured strain. Yielding did not occur in these bars.

For the three reinforced slabs (nos. 13,14 and 15) the maximum load P_{test} was reached at a quite large deflection. The mean value of w_o in Table 2 is 1.21 times the slab thickness for these slabs. For the three unreinforced slabs this factor is only 0.33. When the load was 0.5 P_{test}, corresponding to some serviceability limit, the deflection with an allowance for the initial load, was about 0.10 h for the reinforced slabs and 0.02 h for the unreinforced slabs.

All the slabs showed a crack pattern similar to the classical yield line pattern in slabs without horizontal restraints. The reinforced slabs had several fine cracks, the unreinforced slabs a few wide cracks.

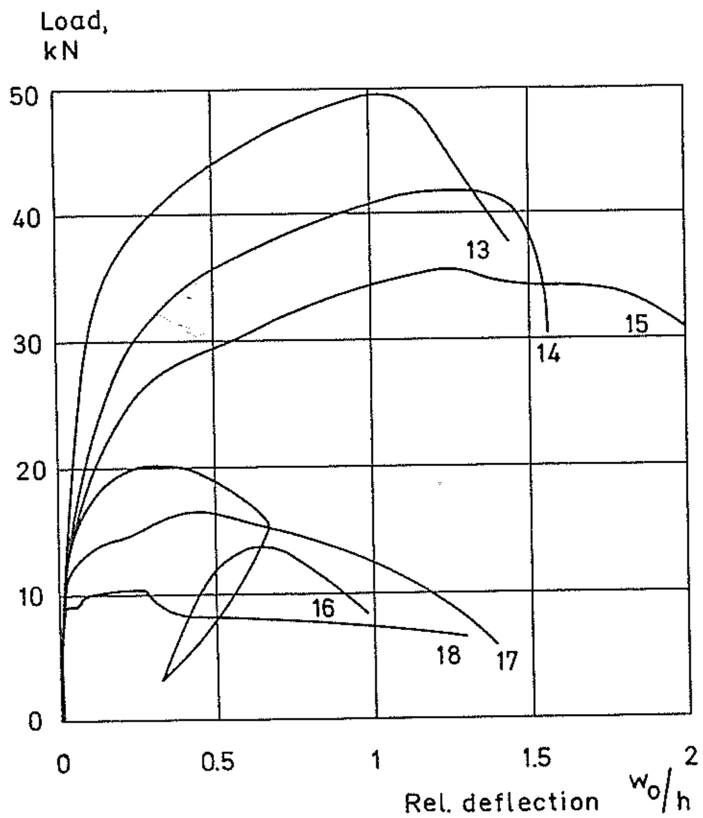

FIG. 3. Test results for slabs no. 13-18.

w_o was measured at the centre of the slabs.

TABLE 2. Measurements taken at max. load P_{test}. u_1, u_2 and u_3 were measured at the points marked 1, 2 and 3 in Fig. 1.

Slab no.	P_{test} kN	Outward movements u_1 mm	u_2 mm	u_3 mm	Force in connecting bars, kN	Vertical deflect. w_o, mm	Relative deflect. w_o/h
13	49.8	1.38	1.23		42	45.1	1.05
14	42.0	1.25	0.72		25	51.6	1.30
15	35.5	1.01	0.95	2.24		46.2	1.29
16	20.3	0.68	0.68		28	12.1	0.29
17	16.4	0.78	0.65		22	16.9	0.42
18	10.1	0.66	0.75	1.71		10.4	0.27

2. IN-SITU TEST

This test is described in other papers /1 and 2/, wherefore only a few facts are given below.

2.1 The slab

The in-situ tested slab, SØ2, was part of a reinforced concrete deck in a factory building under demolition. It was an edge panel, simply supported on a brick wall along one side and surrounded by edge beams and adjacent panels on the other three sides.

The slabs dimensions were l×b×h = 3880×3640×132 mm, which is very close to 3 times the dimensions of slab models. The concrete strength f_{cyl} = 27.7 MPa was obtained from tests on drilled cores.

2.2 The method of testing

Due to the imminent demolition of the building it was decided to use dead load as loading. Blocks of lead each weighing about 1 ton were kindly supplied by Paul Bergsøe & Son Ltd and placed by crane at an attempted mean rate of 4 kN per minute.

During the test vertical displacements were measured by levelling to staffs hanging from the slab.

2.3 The test results

When the load was about equal to the calculated Johansen load 252 kN, the first cracks were observed on the top surface.

When 58 lead blocks were placed on the slab, the various readings were taken, but before block no. 59 could be placed, the slab collapsed. The 58 blocks and the self weight of the slab amounted to 602 kN or 2.4 times the Johansen load.

Also in this slab the crack pattern was similar to the classical yield line pattern.

3. LOAD DUE TO MEMBRANE ACTION

The load due to membrane action is here defined as the maximum load minus the Johansen load

$$P_M^* = P_{test} - P_J$$

However, the values of both P_{test} and P_J depend on the position of the load. They will both be larger, if the load is placed nearer the supports. To eliminate this variation a modified membrane load P_M, corresponding to uniformly distributed load, is defined by

$$P_M = (P_{test} - P_J)\bar{x}_p/\bar{x}_o$$

where $\bar{x}_p$ is the mean value of the distance from the point loads to the nearest support, and $\bar{x}_o$ is the value $\bar{x}_p$ would have had, if the load was uniformly distributed.

In Table 3 are given the values of P_M calculated as described and with

$$\bar{x}_p/\bar{x}_o = 312/208 \quad \text{for the six model slabs and}$$

$$\bar{x}_p/\bar{x}_o = 916/719 \quad \text{for the in-situ slab SØ2.}$$

The Johansen load P_J is here determined using a simple mechanism of collapse with corner yield lines at 45^o to the supports. However, as this leads to an overestimate, the result has been reduced by 10% /1/. The yield moments have been calculated assuming a constant stress in the concrete compression zone equal to

$$f_c = \frac{2}{3} f_{cu} = \frac{2}{3\times0.8} f_{cyl}$$

TABLE 3. Calculated membrane action

Slab no.	P_{test} kN	P_J kN	P_M kN	$\frac{P_M}{h^2 f_{cyl}}$
13	49.8	26.6	34.8	0.90
14	42.0	25.6	24.6	0.95
15	35.5	23.1	18.6	0.82
16	20.3	0	30.4	0.97
17	16.4	0	24.6	1.00
18	10.1	0	15.2	0.54
SØ2	602	252	446	0.92

3.1 Trends in the test results

The two most important parameters for the membrane action are, according to f.ex. Morley /3/, the slab thickness h and the concrete strength f_{cyl}. Dividing the load due to membrane action P_M by $h^2 f_{cyl}$, making it non-dimensional, a remarkably constant result is obtained in Table 3.

The amount of reinforcement, in Table 3 represented by P_J, has apparently no influence on the membrane action.

The effectiveness of the horizontal restraint was expected to be an important parameter. The horizontal restraint was most effective for the slabs nos. 13 and 16 with 4 bars connecting the edge zones, see Fig. 1, and least effective for the slabs nos. 15 and 18 with no connecting bars. The values in Table 3 show that the effectiveness of the horizontal restraint has some influence, but not as much as might be expected.

4. INCLUSION OF OTHER TEST RESULTS

Loading tests on rectangular slabs with horizontal restraint on three sides performed in other laboratories are treated below.

4.1 Details of the tests

Hopkins and Park /4/ tested a model of a floor system consisting of 3×3 rectangular slabs loaded by water bags. In Table 4 is given the result of the testing of an edge slab. The Johansen load P_J is calculated with $d = d' = 0.73$ h and with hogging moment also on the edge without adjacent panel, as the edge beam was designed to carry the induced torsion. P_J has been reduced by 10% as only a simple mechanism of collapse is used.

Park /5/ tested in 1971 two adjacent square slabs using pressure bags. Three edges of each slab were clamped and with horizontal restraints. An edge beam on the fourth side was designed to carry the torsion induced by the hogging moment in the slab. P_J is calculated with $d = d' = 0.78$ h and the 10% reduction.

Hung and Nawy /6/ tested 5 square slabs and 4 rectangular slabs using pressure bags. All the slabs had three edges clamped and with horizontal restraints. The fourth edge was simply supported and without horizontal restraint. P_J is calculated with $d = d' = 0.80$ h and the 10% reduction.

Park /7/ tested in 1964 8 rectangular slabs using pressure bags. All the slabs had three edges clamped and with horizontal restraints. The fourth edge was simply supported on rollers. P_J in Table 4 is 0.9 times the value calculated by Park. $A_s f_y/(h f_{cyl})$ is obtained from values of $T = A_s f_y$ given by Park.

TABLE 4. Details of 26 tests with restraints on three sides. The length of the fourth side is underlined.

Authority	Slab mark	Dimensions l x b x h mm	$\frac{l+b}{2h}$	f_{cyl} MPa	$10^3 A_s f_y/(hf_{cyl})$ Short span Bottom	Short span Top	Long span Bottom	Long span Top	$\frac{P_{test}}{h^2 f_{cyl}}$	$\frac{P_J}{h^2 f_{cyl}}$	$\frac{P_M}{h^2 f_{cyl}}$
Hopkins and Park	B	1588x1130x49	27.7	30.0	18	20	17	21	1.15	0.61	0.54
Park 1971	1	1447x1447x39.1	37.0	26.5	26	26	26	26	1.62	0.87	0.75
Hung and Nawy	C2-1	1524x1524x63.5	24.0	38.7	56	56	56	56	2.02	1.62	0.40
	2			38.7	28	28	28	28	1.63	0.83	0.80
	3			42.1	33	33	33	33	1.76	0.97	0.79
	4			42.1	34	34	34	34	2.01	1.01	1.00
	5			39.2	34	34	34	34	1.98	1.00	0.98
											0.79
	C5-1	1524x1067x63.5	20.4	27.2	51	51	51	51	2.09	1.52	0.57
	2			27.2	40	40	40	40	1.93	1.20	0.73
	3			34.3	63	63	63	63	2.33	1.87	0.46
	4			34.3	39	39	39	39	2.12	1.17	0.95
											0.68
Christian-sen and Frederiksen	13	1250x1250x43	29.1	20.9	37	37	37	37	1.93	1.03	0.90
	14	40	31.3	16.1	52	52	52	52	2.44	1.49	0.95
	15	36	34.7	17.5	53	53	53	53	2.35	1.53	0.82
	16	42	29.8	17.8	0	0	0	0	0.97	0	0.97
	17	40	31.3	15.3	0	0	0	0	1.00	0	1.00
	18	39	32.1	18.6	0	0	0	0	0.54	0	0.54
			31.4								0.86
	SØ2	3880x3640x132	28.5	27.7	33	17	26	13	1.59	0.67	0.92
Park 1964	B-1	1524x1016x50.8	25.0	24.7	22	45	22	45	3.38	1.05	2.33
	2			25.1	43	86	21	42	3.97	1.65	2.32
	3			26.0	80	160	20	41	4.07	2.58	1.49
	4			26.3	111	222	18	37	5.18	3.17	2.01
											2.04
	C-1	1524x1016x50.8	25.0	34.6	15	30	15	30	2.00	0.61	1.39
	2			30.4	35	71	17	34	2.56	1.08	1.48
	3			28.1	73	146	18	37	2.66	1.76	0.90
	4			35.3	86	172	15	29	3.12	1.83	1.29
											1.26

4.2 Conclusions

Fig. 4 shows the load due to membrane action as a function of the slab slenderness defined as average span over thickness. Only the tests by Park 1964 give values of $P_M/(h^2 f_{cyl})$ greater than 1.0, and this is most likely caused by the very rigid steel frame used in these tests in order to effectively prevent outward movement of the slab edges.

The 18 slabs with test frames, which are comparable with conditions in practice, give values of $P_M/(h^2 f_{cyl})$ between 0.40 and 1.00. Using mean values of each of the six test series, the interval is 0.54 to 0.92. With a characteristic value equal to 0.70, the estimate

$$P = P_J + 0.70\ h^2 f_{cyl}$$

is obtained for the bearing capacity of a square or rectangular, reinforced or unreinforced concrete slab with normal horizontal restraints on three of the four sides and with the load P uniformly distributed.

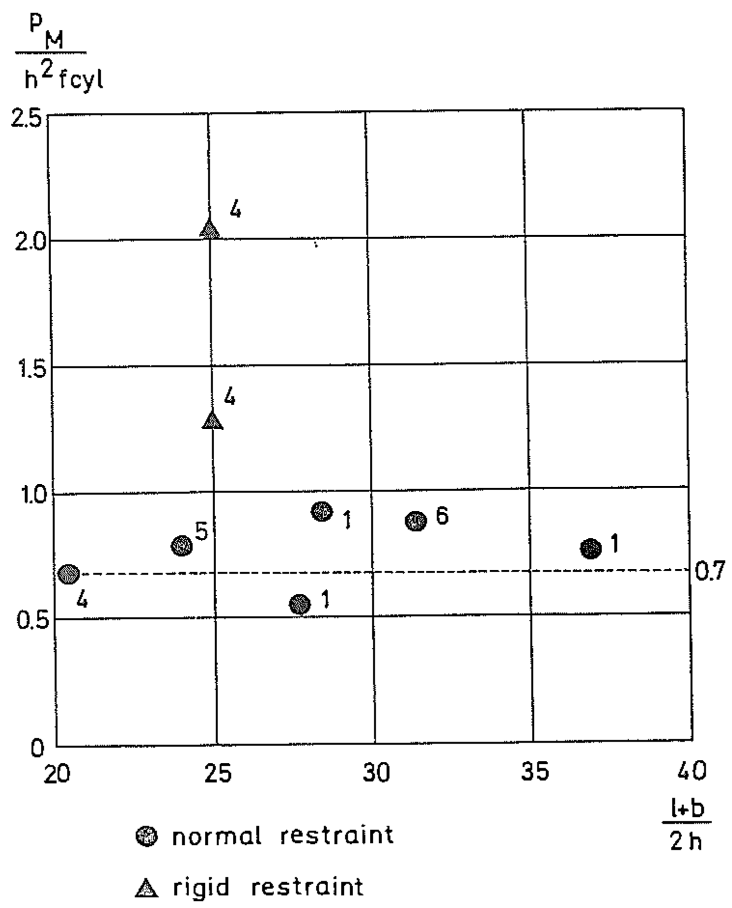

FIG. 4. Membrane action versus slenderness.

The numbers next to the points are the numbers of tests represented.

It should be noted that the two smallest membrane loads shown in Fig. 4 are from rectangular test slabs, where one of the longer edges was unrestrained against horizontal movement. Therefore a factor smaller than 0.70 should be used in the expression, when the side ratio l/b is 1.5 or greater and one of the longer sides horizontally unrestrained.

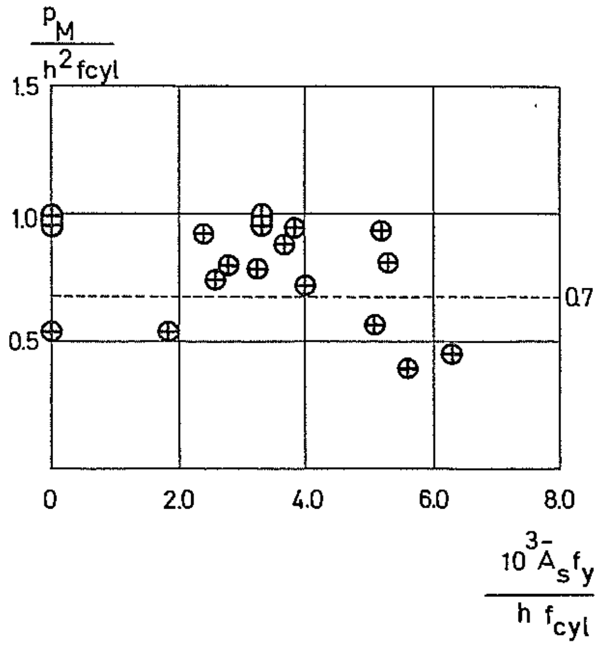

FIG. 5. Membrane action versus reinforcement

for 18 tests with normal restraint. $\bar{A}_s$ is the mean of A_s and A'_s with allowance for simply supported edges.

Finally Fig. 5 shows that the amount of reinforcement actually has little or no influence on the load due to membrane action, at least as long as the slab is not heavily reinforced. This fact is utilized in the expression above, where the steel areas only govern the Johansen load P_J.

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